Paraxial ray optics cloaking

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Despite much interest and progress in optical spatial cloaking, a three-dimensional (3D), transmitting, continuously multidirectional cloak in the visible regime has not yet been demonstrated. Here we experimentally demonstrate such a cloak using ray optics, albeit with some edge effects. Our device requires no new materials, uses isotropic off-the-shelf optics, scales easily to cloak arbitrarily large objects, and is as broadband as the choice of optical material, all of which have been challenges for current cloaking schemes. In addition, we provide a concise formalism that quantifies and produces perfect optical cloaks in the small-angle ('paraxial') limit.

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INTRODUCTION

Much scientific interest and progress have been developed in invisibility cloaking since the seminal works of Leonhardt [1] and Pendry, Schurig, and Smith [2]. They provided a theoretical framework to create a curved space for light waves, by carefully constructing materials through a transformation of space. This new field of research has been called 'transformation optics.' [3] Experimental realization of these ideas has been difficult, due to the need for artificial electric and magnetic materials (called 'metamaterials'), its narrow-band spectrum, infinite phase velocity (or negative index to compensate this), and anisotropy in the theory. [4] Nonetheless, there has been tremendous progress in scientific cloaking, inspired by transformation optics. This includes a 'carpet cloak' [5] and even cloaking in time. [6] A few groups have been able to cloak mm to cm size macroscopic objects as well, using birefringent materials. [7, 8]

To overcome the metamaterial requirements and to extend cloaking to a broadband, visible regime, researchers have recently looked to geometric optics for cloaking. [9, 10] In these works, only the amplitude and direction of light fields are considered, as opposed to the full preservation of fields (amplitude *and* phase) of transformation optics. These designs have been able to cloak up to meter-sized objects [9, 10] with commonly available optics. Yet, these schemes work for unidirectionally incident light, or discrete directions, and are not designed for continuous multidirectional cloaking. This can be seen in Figure 1 of Reference 10, where rays that go through the center at non-zero angles can actually enter the cloaking region, effectively uncloaking the region.

We take a slightly different approach than transformation optics. Rather than bending or reshaping the space of the fields, we investigate the possibility of altogether eliminating the cloaked space or replacing it entirely. To be clear, our scope is limited to ray optics only. We do not attempt to preserve the phase, for which much elegant work continues to be done.

DEFINING A 'PERFECT' CCOAK

We first clarify what we mean by a "cloak" in this paper. Rather than the commonly used definition of a wearable clothing, we use the other definition of "cloak," which is to "hide." In this regard, our use of "cloaking" includes a broad meaning of "invisibility."

Now, let's suppose we have a black box that is to hide an object in its space. Let's first discuss what a 'perfect,' or 'ideal,' cloaking box would do to light rays as seen by an observer. An obvious first requirement is that any cloaking system must have a non-zero volume to hide an object. In addition to this, we may initially try to make a box such that light does not 'see' it. Thus, we would attempt to remove the space of the box. If this is done, then light rays entering the box should exit the box exactly as it entered. This is called an "identity transformation." Just like particles are described by their positions and momenta, light rays can also be described by their positions and angles of direction. So for a box that is an identity transformation, the positions and angles of the light rays will not change (See Figure 1 (a)).

What is the physical effect of removing the box volume? The exiting rays, as seen by an observer, will be closer, by the length of the box (See Figure 1 (b)). Thus, an object behind the device will appear to be closer than its actual location. The mirror device in Figure 9 of our paper [10] is a box that is similar to the identity transformation, but it *adds* distance instead, making the object behind it to appear *farther* away.

The identity transformation is a good candidate for a cloaking box. It is not perfect, of course. If we did have a perfectly cloaking box, then it should make objects appear exactly where they are. In fact, a perfect cloak should act the same way as if it was filled with the surrounding medium. So to cloak in air, the black box should act like it *was* air (See Figure 1 (c)). This is then a sufficient and necessary condition for defining a *'perfect cloak'* for ray optics- it behaves as if its space was replaced by the surrounding medium, for all light rays entering it. In fact, Leonhardt had stated just such a definition for a perfect invisibility device [1].

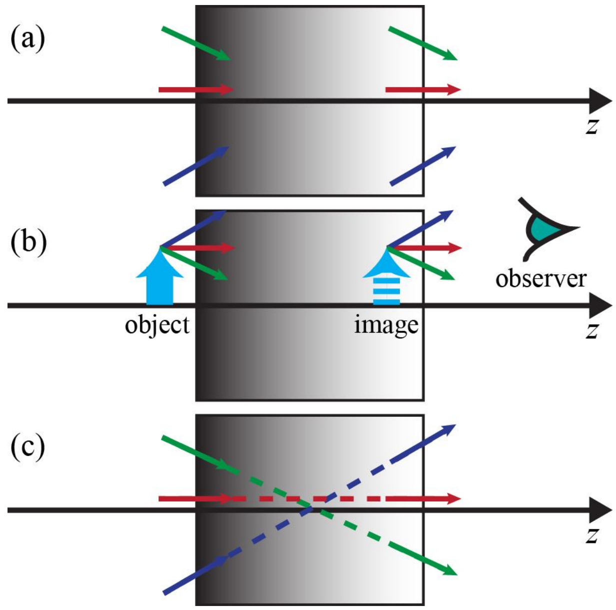

FIG. 1. Possible candidates for ray optic cloaking. (a) A black box that is an identity transformation. Three rays of light enter the box from the left. They exit (to the right) with the same positions and directions. (b) How an observer views an identity transformation box. An object behind the box will appear to be closer than its real position, by the length of the box. (c) A black box that is a 'perfect' cloak. Rays exit the box as if the box was filled with the surrounding medium. Angles do not change, but the positions shift proportionally to the ray angles and box length.

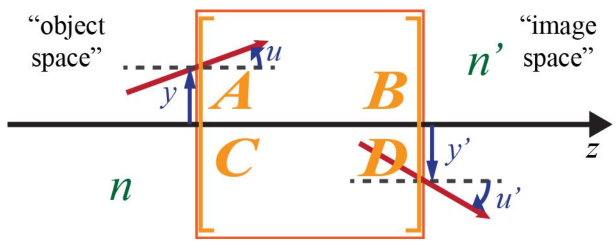

FIG. 2. Defining light rays and the 'ABCD' matrix in the paraxial approximation, for a rotationally symmetric system (about the z-axis). The optical system (box in center) can be described by an 'ABCD' matrix. This matrix maps the initial position (y) and angle (u) to those exiting the system (y', u'). The "object space" is the space before the ABCD system, with index of refraction n. Likewise, the "image space" is the space after the system, with index of refraction n'. In this diagram, $y > 0$, $u > 0$, $y' < 0$, and $u' < 0$, with our sign convention [11].

ABCD MATRIX

We will now develop a simple formalism to precisely quantify and define our 'perfect cloak.' We will borrow well-known linear equations from the field of geometric optics. To first approximation, called the "paraxial approximation," light rays are assumed to deviate minimally from the axis of rotational symmetry for the system (z in Figure 2). We can then see that in the paraxial approximation, ray angles are small.

All light rays can then be described by its position y and the paraxial angle u, much like an object is described by position and momentum. This assumes that the system is rotationally symmetric about the z-axis. Here

$$u \equiv \tan\theta \approx \theta, \tag{1}$$

where θ is the exact, real angle of the ray from the $z-$axis.

Because of the linearity of optics in the paraxial approximation, the propagation of light rays through an optical system can be described by matrices. These matrices are called 'ray tracing' matrices, or 'ABCD' matrices, and are used as follows:

$$\begin{bmatrix} y' \\ n'u' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y \\ nu \end{bmatrix}, \tag{2}$$

where n and n' are the indices of refraction for the space before and after the ABCD matrix, respectively (See Figure 2). For example, the ABCD matrix for a space of length t, with index of refraction n_t, is the 'transfer' matrix [12]

$$M_t = \begin{bmatrix} 1 & t/n_t \\ 0 & 1 \end{bmatrix}. \tag{3}$$

METRICS TO MEET FOR A PERFECT CLOAKING SYSTEM

So what does the ABCD matrix for a perfect cloaking system look like? It is precisely the transfer matrix M_t in equation (3), where $t = L$ is the length of the system, and $n_t = n = n'$. This is because a perfect cloaking device simply replicates the surrounding medium throughout its volume. We then see that

$$\begin{bmatrix} y' \\ nu' \end{bmatrix} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ nu \end{bmatrix} = \begin{bmatrix} y + Lu \\ nu \end{bmatrix} \tag{4}$$

As expected, the angle remains the same, i.e. $u' = u$, and the position shifts by the angle multiplied by the length, i.e. $y' = y + Lu$.

We have then provided a requirement for a perfect optical cloaking system that applies to all rays, within the first order approximation. Any cloak that can be considered 'perfect' should also be so in this first order approximation, so this is a necessary condition for *all* orders. To be clear, it is only a sufficient condition within the first order paraxial approximation and not for higher orders. Note that this does not violate the findings by Wolf and Habashy [13] and Nachman [14] since this is a paraxial approximation, and hence does not work for large angles. However, we show this to be a surprisingly effective condition, despite its simplicity. For clarity, we restate this requirement:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{perfect cloak}} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}, \tag{5}$$

where L and n are the length of the cloaking system and index of refraction of the surrounding medium, respectively.

Because ABCD matrices have a determinant of 1, equation (5) gives only three conditions to be satisfied:

$$B = L/n\,,\ C = 0\,,\ \text{and}\ (A = 1\ \textit{or}\ D = 1). \tag{6}$$

Note that a perfect cloaking system is "afocal" ($C = 0$), meaning the optical system has no net focusing power, so an object at infinity will be imaged to infinity. This is helpful when designing, since an afocal condition can be easily checked.

Designing a perfect paraxial cloak with rays

It may not be obvious that an optical system can satisfy equation (5), despite containing a cloaking region. The discussion and conditions for a 'perfect cloak' may have little meaning unless a physical solution actually does exist. We will now build general optical systems, to see whether a perfect paraxial cloak can be designed with rays. We attempt to find the simplest nontrivial solution, so we will only consider rotationally symmetric systems with thin lenses, and in free space with $n = 1$.

The ABCD matrix for one thin lens is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{thin lens}} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}, \tag{7}$$

where f is the focal length of the lens. We can easily show that equation (5) will only be satisfied by the one or two lens cases only if the lenses have no optical power. This has no cloaking region and no optical effect. For a three lens system, equation (5) requires it to be a two lens system, so it cannot be a perfect cloak. However, it can asymptotically approach a paraxially perfect cloak.

A minimum of four lenses are required to satisfy equation (5) with thin lenses only. We desire to undo any changes that the first half of our system makes, as a possible strategy to make the system behave as if absent. This can be done by making the second half symmetric to the first half ($f_1 = f_4$, $f_2 = f_3$, $t_1 = t_3$). We then require $A = 1$ and $C = 0$ for such a four lens ABCD matrix. Both conditions are simultaneously satisfied by

$$t_1 = f_1 + f_2. \tag{8}$$

With equation (8), we then set $B = (2t_1 + t_2)$, and solve for t_2:

$$t_2 = 2f_2(f_1 + f_2)/(f_1 - f_2). \tag{9}$$

The total length of the system is then

$$L = 2t_1 + t_2 = 2f_1(f_1 + f_2)/(f_1 - f_2). \tag{10}$$

Although these solutions satisfy equation (5) mathematically, checks must be made to ensure they contain a finite cloaking region and are physically feasible.

EXPERIMENT AND SIMULATION: A PARAXIAL FOUR LENS CLOAK

We now simulate a four lens 'perfect' paraxial cloak for our experimental setup, that has symmetric left and right halves. Real lens systems produce aberrations that can blur and distort the observed image. So we used 'achromatic doublets' that combine two lenses as one, to correct for chromatic (color) and other aberrations. We corrected equations (8) and (9) to include the lens thicknesses, and calculated t_1, t_2, and t_3 ($t_1 = t_3$). The simulation in Fig. 3 show rays with a field-of-view of about 3 degrees. The cloaking region is an elongated cylinder between the lenses where the rays do not pass. The system is easily scalable. For example, we only need to double all radii of curvature, lengths, and entrance pupil to obtain double the cloaking space in each dimension.

FIG. 3. **CODE V simulation of a symmetric, perfect paraxial cloak, with four lenses using rays.** Four achromatic doublets are placed with separations determined from equation (5). Entrance pupil is 50 mm, with $-1.5°$ to $1.5°$ field-of-view. Simulations are shown with *no* separate optimization. Object is placed at infinity.

In constructing our four lens cloak, we used achromatic doublets to reduce the aberrations of the images. Photographs of this paraxial cloak are shown in Fig. 4. The grids on the wall were 1.9 m from the closest lens to the back. The camera was 3.1 m away from the front lens, but optically zoomed in by 21x. The images were taken from $-0.65°$, on-axis ($0°$), at $0.47°$, and $0.95°$ viewing angles, by changing the height of the camera. A ruler was placed behind the second doublet from the front. The middle of the ruler is cloaked near the center-axis of the device. In particular, the grids on the wall are clear for all colors, have minimal distortion, and match the sizes and shifts of the background grids for all the angles, demonstrating the quality of this multidirectional cloak.

CONCLUSION

In summary, we have defined what a perfect cloak should do in ray optics. We then provided a sufficient and necessary algebraic condition for a perfect cloak in the first-order, or paraxial, approximation. We finally

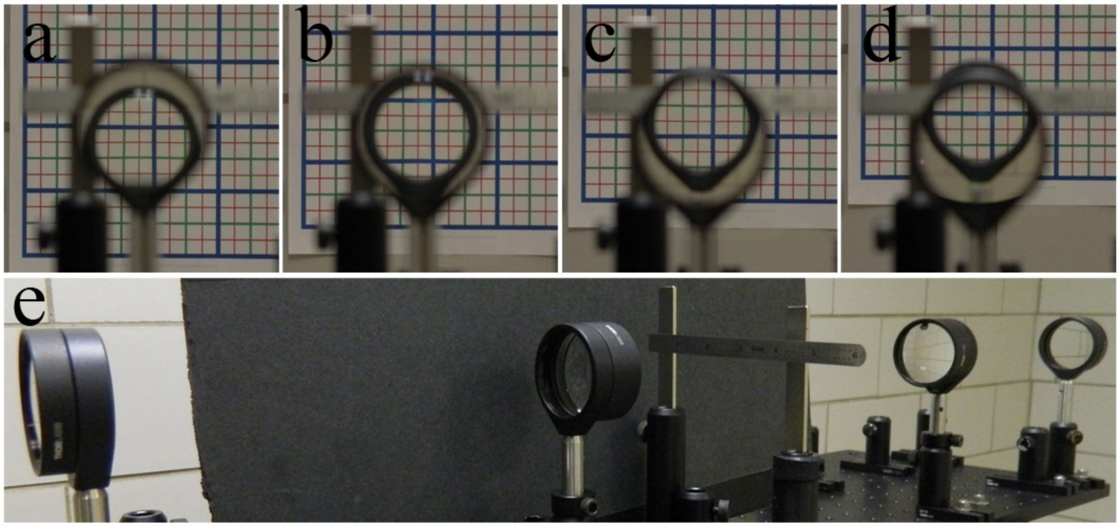

FIG. 4. **Experimental demonstration of a 'perfect' paraxial cloak with four lenses.** Camera was focused on the wall. The grids on the wall can be seen clearly, and match the background for all colors and viewing angles. The middle of the ruler is cloaked inside the lens system for all angles shown. Images at various camera-viewing angles: **(a)** $-0.65°$, **(b)** on-axis (0°), **(c)** 0.47°, **(d)** 0.95°. **(e)** Side profile of experimental setup.

derived a device that fits this definition, and experimentally demonstrated it for continuous ranges of directions. Transformation optics and quasiconformal mapping are general formalisms used for cloaking fields. Here we provided another formalism that can effectively describe ray optics invisibility.

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