

## Abstract

- Tang and Cohen [2,3] proposed and demonstrated “superchiral” electromagnetic fields to enhance optical enantioselectivity, typically a very small signal.
- We explicitly show the limitations of such enhancement by including the magnetic susceptibility term.
- We also generalize the dissymmetry factor and Lipkin’s “Z<sup>000</sup> zilch” (or “optical chirality”) to that for a linear medium.

## Definitions

- Optical Enantioselectivity** =

Ability to selectively excite one form, over the other, of a “chiral” molecule (having left- and right-handed forms) using optical fields.

- Dissymmetry Factor** :

$$g = \frac{A^+ - A^-}{(A^+ + A^-)/2} \leq 2, \quad (1)$$

where  $A^\pm$  is Absorption Rate for left(+)- or right(-) handed fields. This is a good measure of optical enantioselectivity.

- Circular Dichroism** =

Numerator of  $g$  ( $A^L - A^R$ ), when Left- and Right-Circularly Polarized Light (LCPL, RCPL) used (aside from some constants).

When CPL used, we denote  $g \rightarrow g_{\text{CPL}}$ .

- Optical Chirality** :

$$\mathcal{C} \equiv \frac{\epsilon_0}{2} \mathbf{E} \cdot \nabla \times \mathbf{E} + \frac{1}{2\mu_0} \mathbf{B} \cdot \nabla \times \mathbf{B}. \quad (2)$$

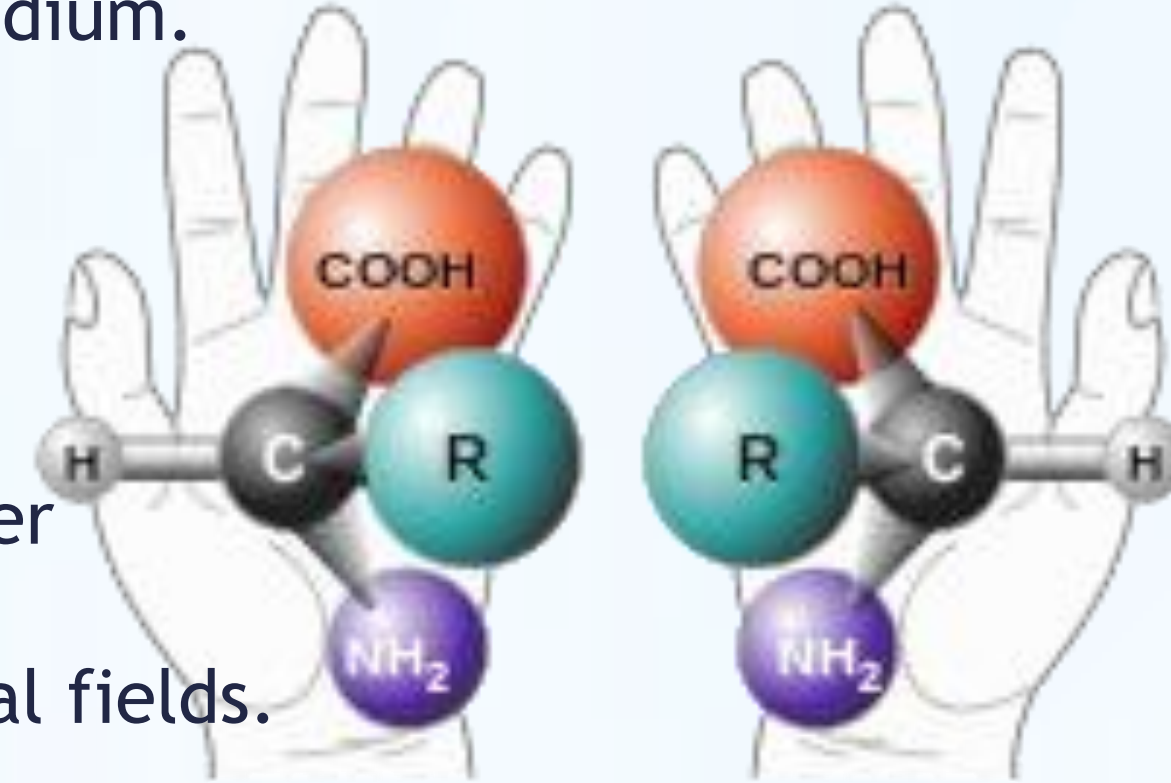
- Defined originally by Lipkin (Z<sup>000</sup> “zilch”) in 1964, this and other zilches are conserved quantities in vacuum, but were dismissed as physically irrelevant [4].
- Tang and Cohen related this to the chiral asymmetry of the exciting fields, and termed it “Optical Chirality” [3].
- Others have since interpreted Lipkin’s zilches to represent the “angular momentum” of the curl of the electromagnetic (E&M) field [5]. A “helicity density,” not optical chirality, has dimensions of angular momentum per unit volume. For monochromatic fields, helicity densities and zilches are proportional by square of the angular frequency.

- ❖ We generalize this vacuum  $\mathcal{C}$  with  $\epsilon_0 \rightarrow \epsilon$  and  $\mu_0 \rightarrow \mu$  for chiral linear media [1].

This implies that  $\mathcal{C}$  is no longer a purely E&M field property.

Also, despite the chiral nature of the new  $\mathcal{C}$ ,

$|g|$  is symmetric with respect to left- or right-handed forms.



## Background

- Tang and Cohen’s theory for enhancement in dissymmetry factor [2,3]:

$$g/g_{\text{CPL}} = \frac{cC}{2U_e\omega}, \quad (3)$$

$c$  = speed of light,  $U_e$  = electric field energy density,  $\omega$  = frequency.

- Enhancement is field-dependent only.
- Material property in  $g_{\text{CPL}}$  only.
- For a standing wave chiral field (called “superchiral” field), at electric field node (Fig. 1):

$$g/g_{\text{CPL}} = \frac{1+\sqrt{R}}{1-\sqrt{R}}, \quad (4)$$

$R$  = reflectivity of mirror used.

- As  $R \rightarrow 1$ , enhancement  $\rightarrow$  Infinity! Authors state magnetic field limits this, but claim 400-fold enhancement w/ large  $R$ .
- Experimental demonstration of ~11x enhancement w/  $R=0.72$  (Fig. 2).

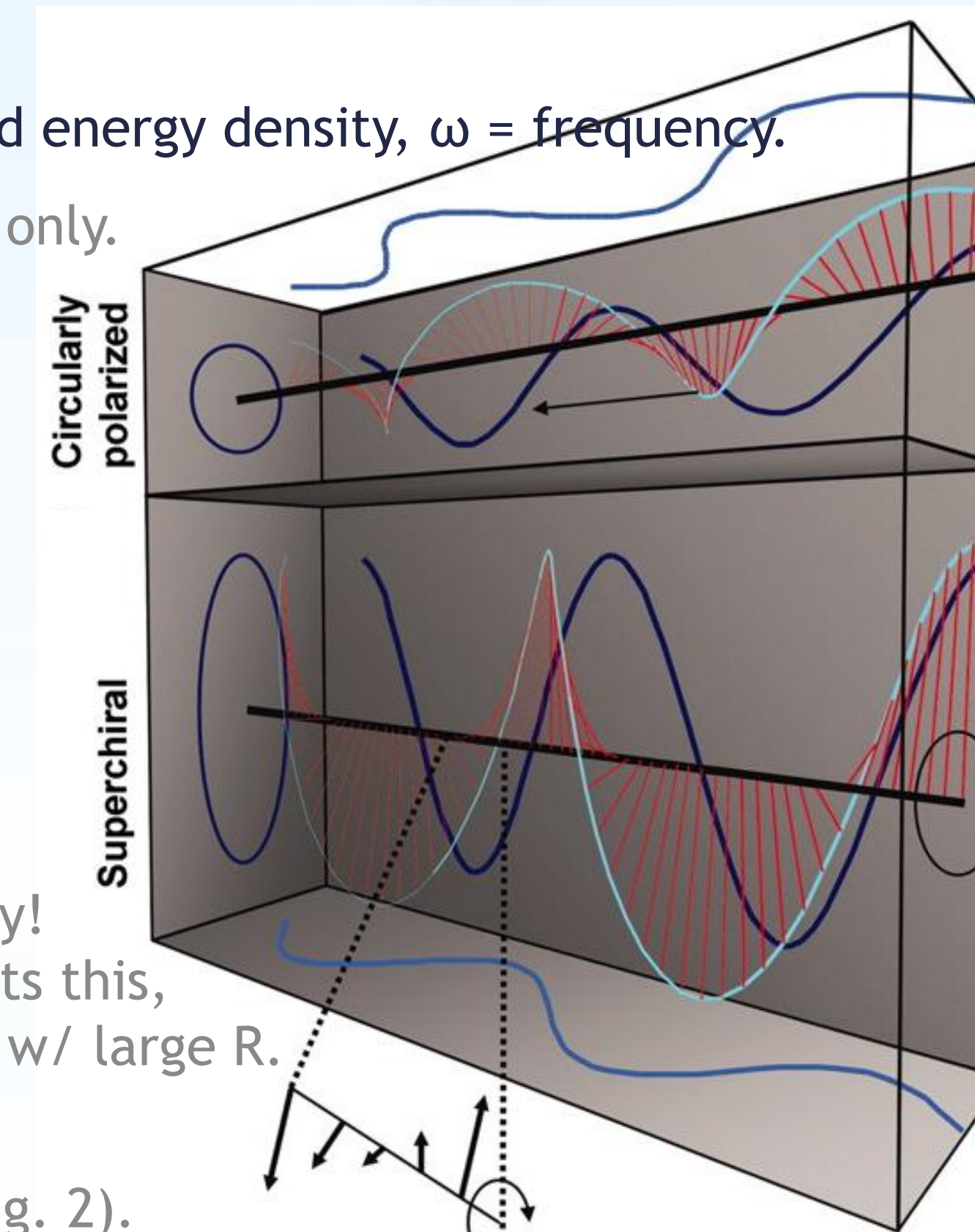


Figure 1: CPL vs. “Superchiral” Field (From [2])

- 10<sup>6</sup> Enhancement w/ 2D Nano-Crosses

- Independently by Hendry et al, shortly after [6].

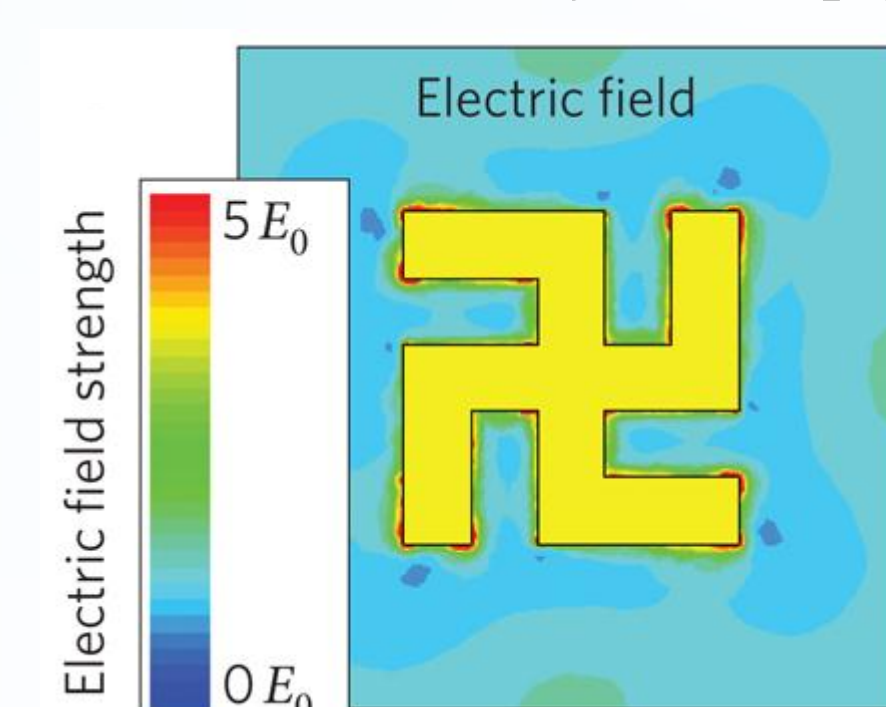


Figure 3: Nano-cross (From [6])

- Enhancement in chiral optical rotation. Used Optical Chirality to explain 10<sup>2</sup> enhancement, but other 10<sup>4</sup> appears to be evanescent near-field effects [7].

- Significance [8]

- Seemingly arbitrarily large enhancements in small chiral optical signals ( $g_{\text{CPL}} \approx 10^{-3} \sim 10^{-2}$ ) [1].
- Possible by engineering E&M fields, independent of chiral molecules.
- Renewed interest in, now physically significant, Lipkin’s zilches.

## Assumptions

- ❖ Since enhancement at electric field node of standing wave, magnetic field is maximum, due to conservation of energy. Though usually small, magnetic susceptibility can no longer be ignored, as was done in Refs [2,3]. This provides correction and limitation of  $g/g_{\text{CPL}}$ .
- Multipole moment approximation with electric polarizability, magnetic susceptibility, and mixed electric-magnetic dipole polarizability. Absorption rate approximation with electric and magnetic dipoles only.

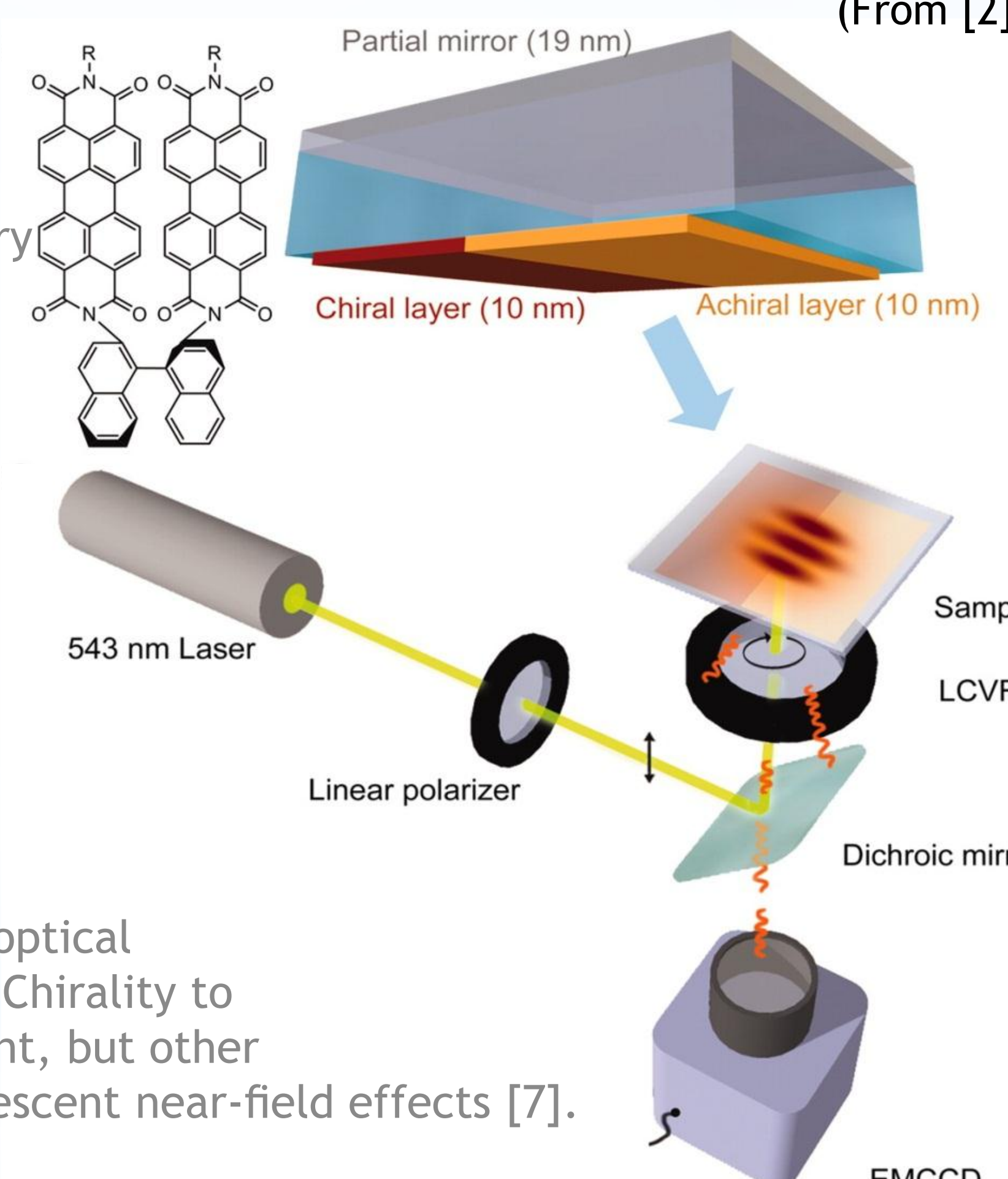


Figure 2: Experimental Setup for “Superchiral” Enhancement (From [2])

## Dissymmetry Factor Corrections

- ❖ Corrected enhancement factor:

$$g/g_{\text{CPL}} = \frac{cC}{2\omega(U_e + \gamma U_b)}, \quad (3')$$

where  $U_b$  = magnetic field energy density,  $\gamma \propto$  (magnetic susceptibility) / (electric polarizability).

- ❖ Correction at electric field node:

$$g/g_{\text{CPL}} = \frac{1-R}{(1-\sqrt{R})^2 + \gamma(1+\sqrt{R})^2}, \quad (4')$$

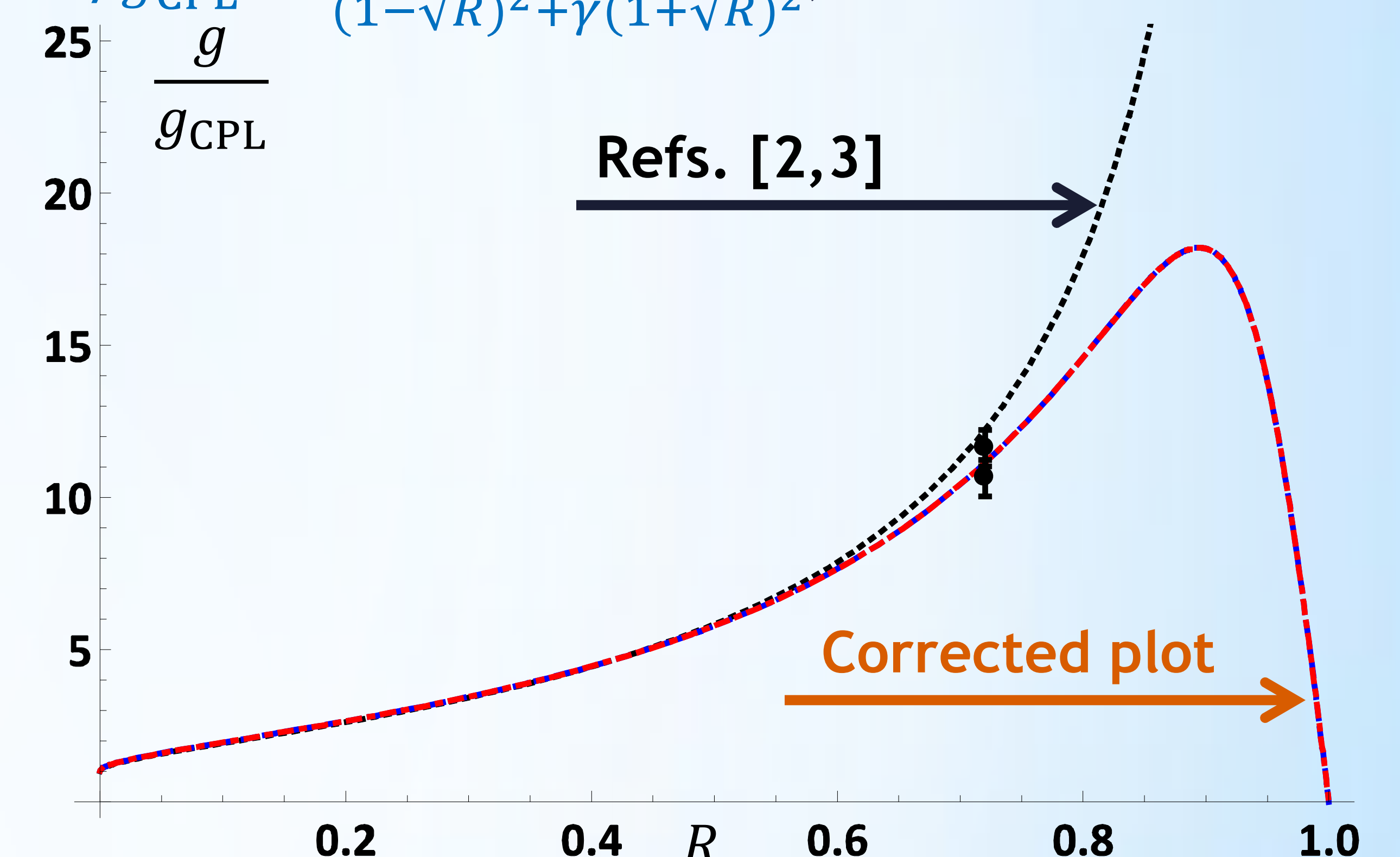


Figure 4: Enhancement in dissymmetry factor vs. reflectivity, for material used in Ref. [2]. Dots w/ error bars obtained experimentally in Ref. [2]. (From [1])

## Limitations

- ❖ Enhancement is finite; < 20x for Ref. [2]; goes to 0 for  $R = 1$  (Fig. 4).
- ❖ For a given material, maximum enhancement is fixed at  $\sim 1/(2\sqrt{\gamma})$ ; for realistic values, the maximum is  $\sim 500x$ .
- ❖ Finding material with small  $\sqrt{\gamma}$  is not practical since signal goes as  $\gamma$  (so signal decreases faster than enhancement factor increase).

## Conclusions

- Cauchy-Schwarz inequality holds for dissymmetry factor when magnetic susceptibility retained.
- Corrected dissymmetry equation (Eqn. (3')) can be helpful for ongoing research and dialogue in nodal enhancement of optical enantioselectivity.

## References

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