



Paraxial Cloaking- A "Rochester Cloak" [1] Is an Ideal, Broadband and Omnidirectional Cloak Possible? [2]

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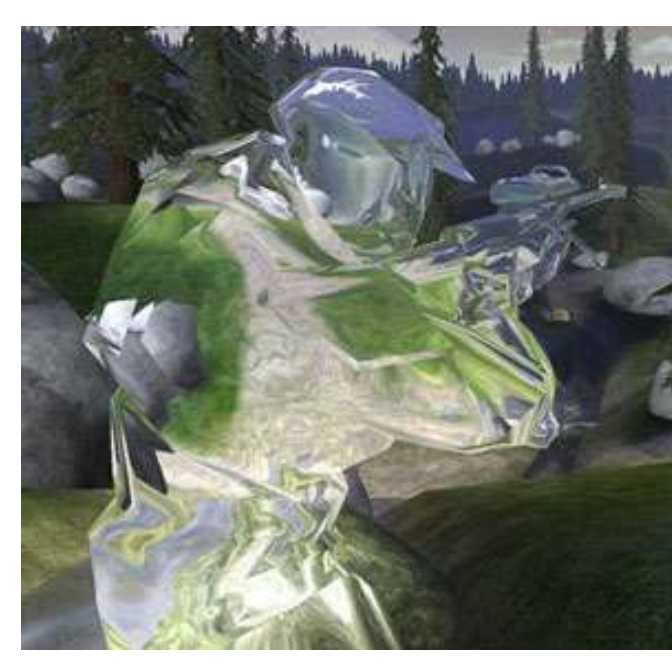


A. Summary

We use 'paraxial' (small-angle) geometric optics to demonstrate a ray optics cloak, after defining invisibility cloaking quantitatively [1]. To our knowledge, our 4 lens design was the first 3D, transmitting cloak that worked for a continuous range of viewing angles, while being broadband for the visible spectrum. We follow with broadband, full-field (phase + amplitude) matching [2].

B. Background

1) Fiction: Harry Potter, Star Trek, games, Invisible Man.

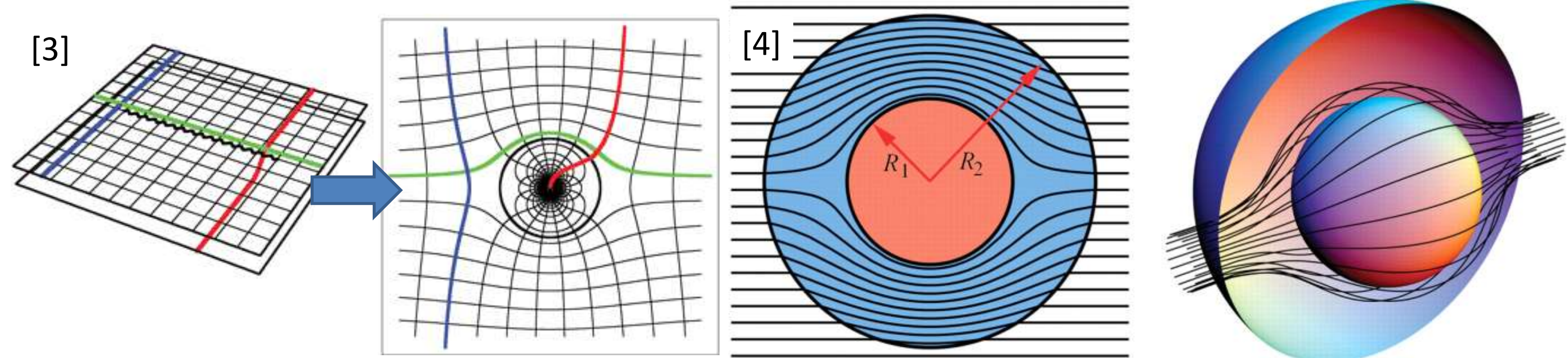


2) Active invisibility: Tachi Lab (Japan, 2003), Mercedes-Benz (2012), Land Rover (2014).



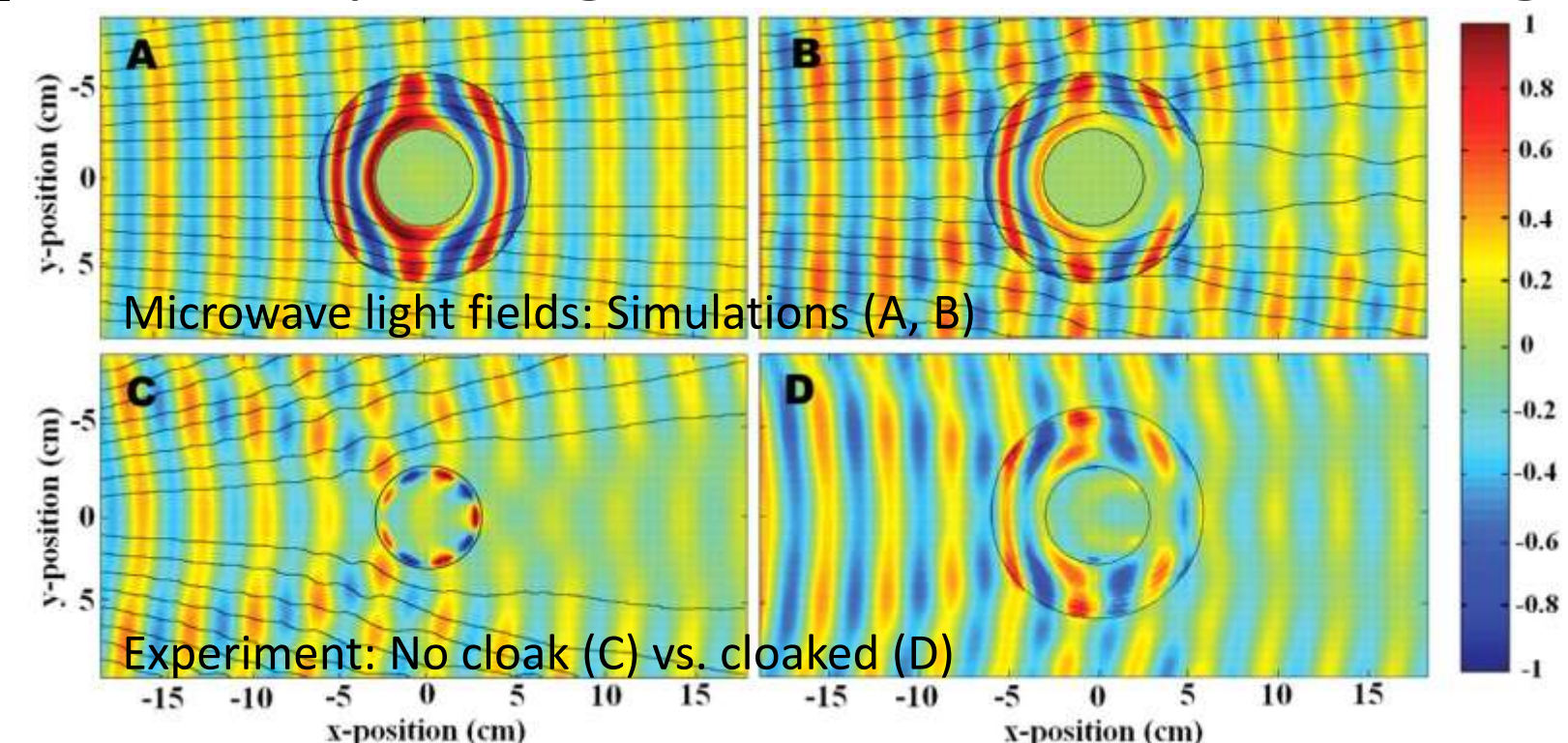
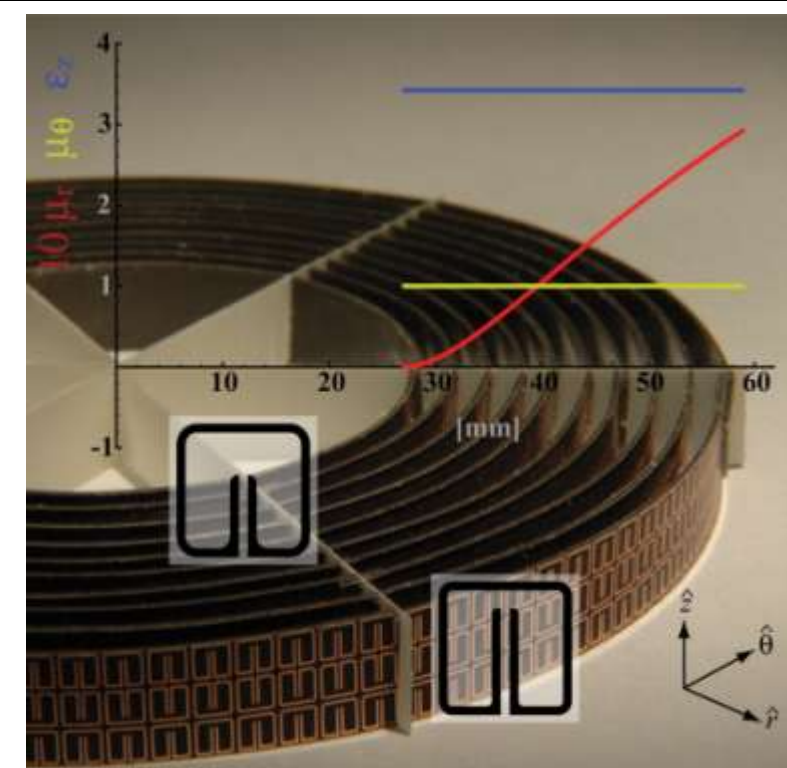
C. Transformation Optics [3,4]

Use coordinate transformations to map a virtual space where no light enters, into physical space where light bends around a real space.



To accomplish this, make 'metamaterials' (artificial materials), that have patterns and electromagnetic properties (ϵ , μ) dictated by the transform.

• **First demonstration (2006)** [5]: Used split-rings for 2D microwave cloaking.



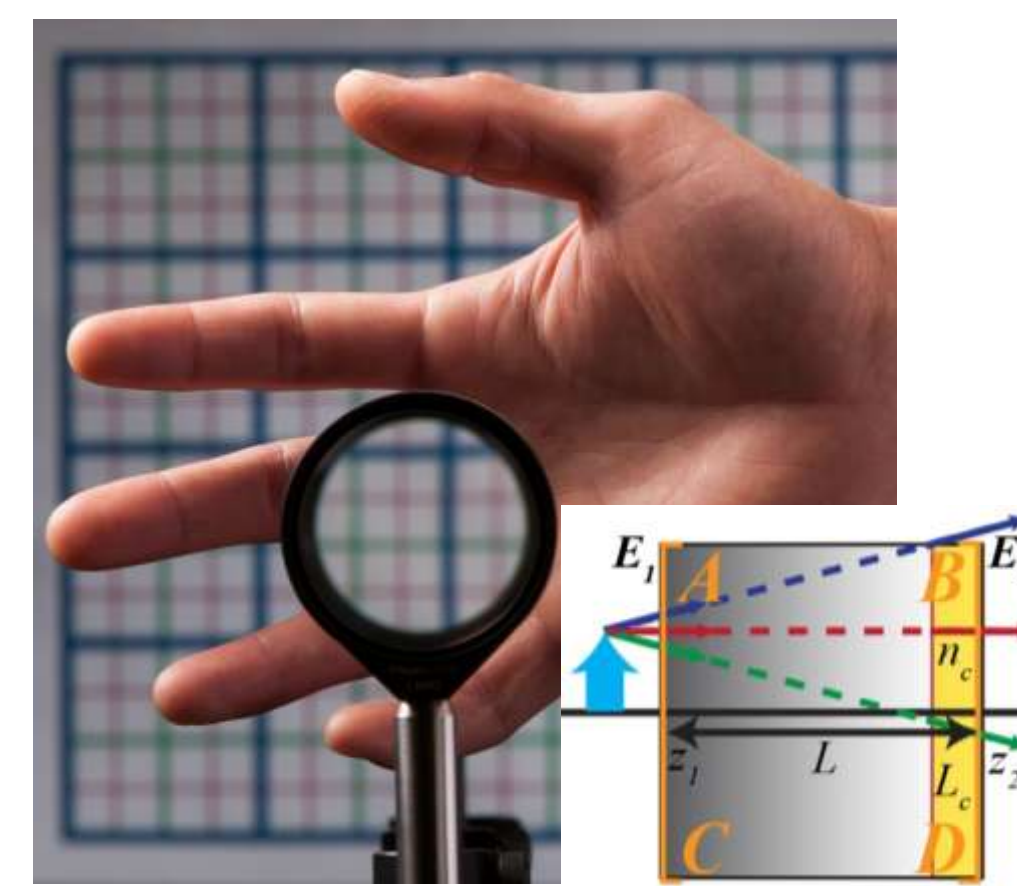
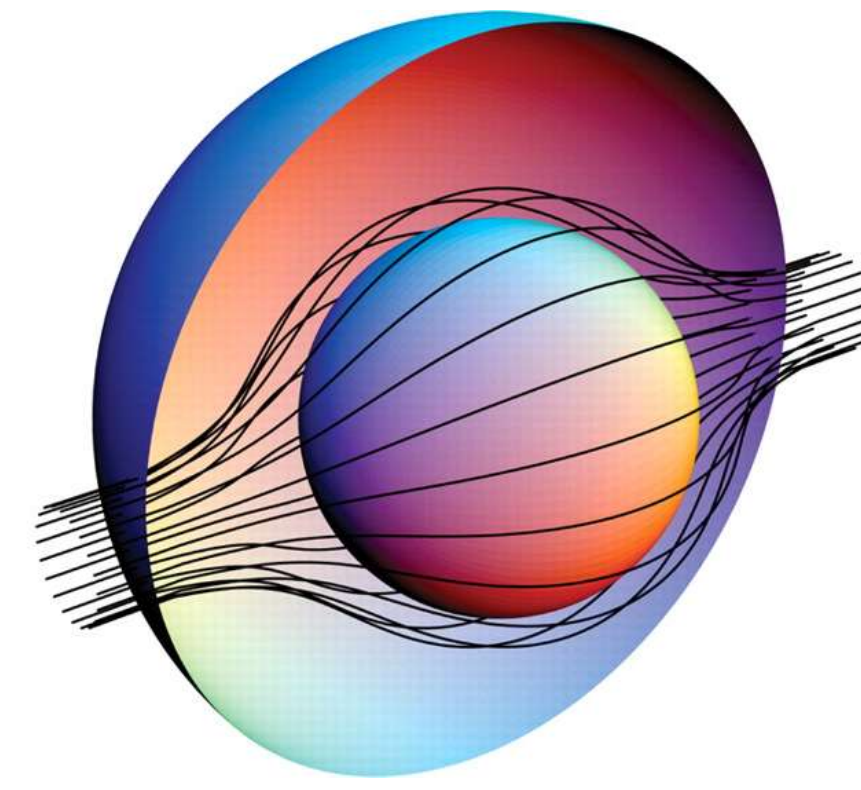
• Great interest in scientific cloaking followed: Temporal event cloaking, thin radio wave canceling cloak, seismic cloaking, 'carpet' cloaking, etc.

D. Define a "Cloak"

• We use the definition "to hide," instead of a wearable clothing.

• A 'perfect' cloak should:

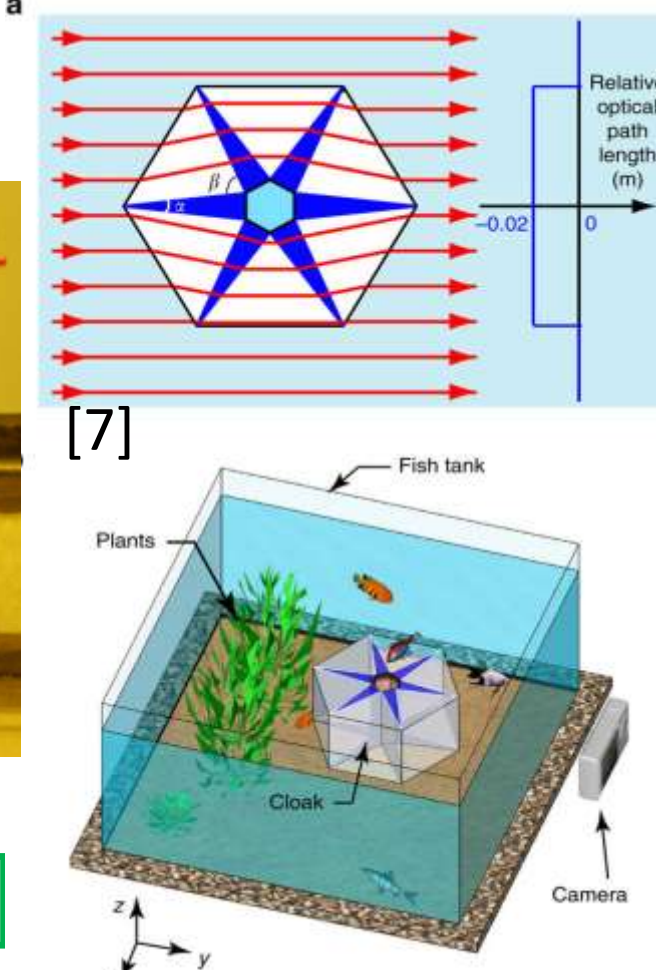
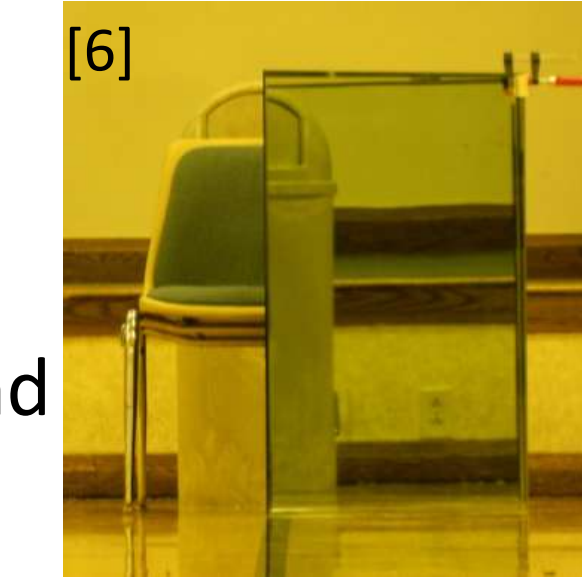
- 1) Cloak a non-zero volume,
- 2) Make itself and the hidden object appear transparent.



| 'Ideal' Cloak Properties | Transformation Optics | Initial Ray Optics Cloaking | Paraxial Cloaking |
|--------------------------|-----------------------|-----------------------------|---------------------------|
| Broadband | Difficult | Excellent | Excellent |
| Visible spectrum | | | |
| Isotropic | | | |
| Macroscopic | | | |
| 3D | Some challenges | | |
| Full-field (+phase) | Excellent | ~No (1 or discrete freq.) | Broadband (theory) |
| Omnidirectional | | 1 or discrete directions | Continuous multidirection |

E. Initial Ray Optics Cloaking [6,7]

- Drop phase-matching of light fields to build macroscopic cloaks for visible light.
- Limited to 1 direction, or discrete directions.
- Issues: For change in viewing angles, background shifts, or cloaking region revealed. Also, some do not match magnification with object.



F. Paraxial Ray Optics Cloaking [1]

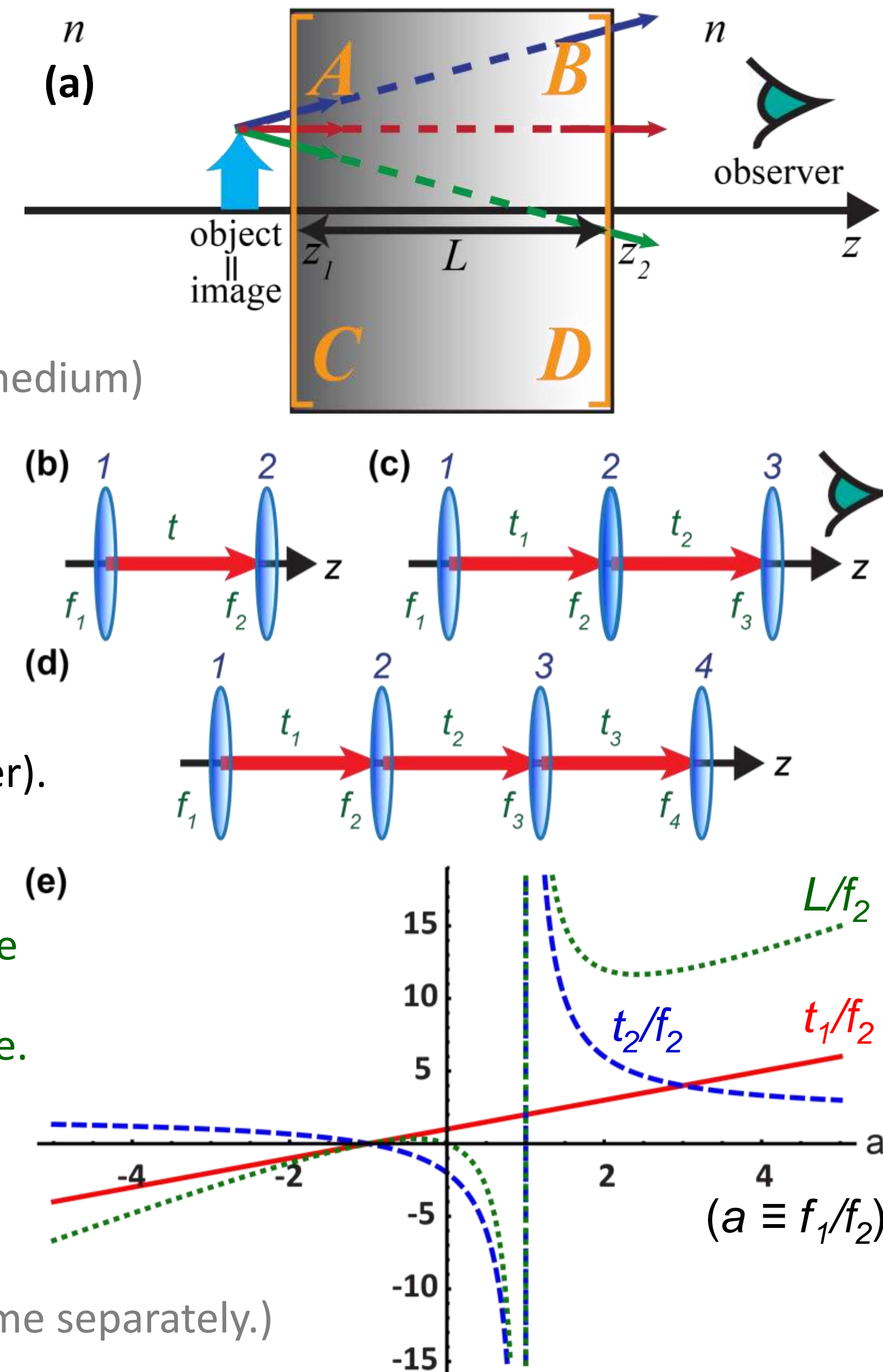
• **Quantitatively define "cloaking"**: Using geometric optics, in the small-angle ('paraxial') limit, a cloak is an optical system with ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{perfect cloak}} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}$$

(L = Length of the cloaking system, n = Refractive index of the ambient medium)

G. Solve Cloaking ABCD Matrix

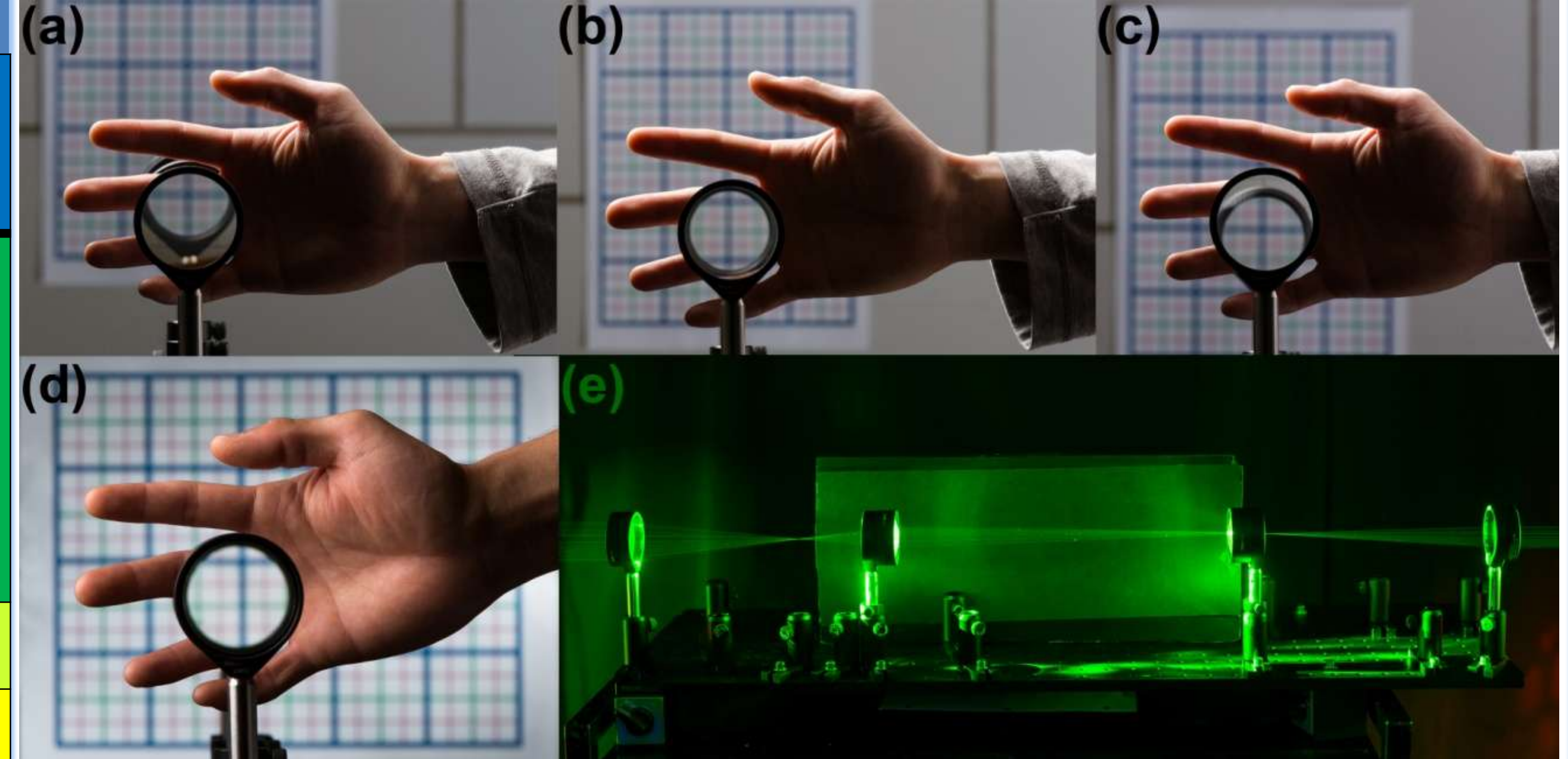
- (a) Find **simplest** ABCD solution: Use rotationally symmetric thin lenses.
- (b) **1 or 2 lens(es)**: $f_1=f_2=\pm\infty$ (no power).
- (c) **3 lenses**: Can asymptotically approach 'perfect' cloak for $L \rightarrow 0$.
- (d) **4 lenses**: At least 4 needed to solve 'perfect' cloak ABCD with lenses and have non-zero cloaking volume.
- (e) Figure (e) shows all left-right symmetric ($f_1=f_4, f_2=f_3, t_1=t_3$) 4 lens cloaking solutions*.



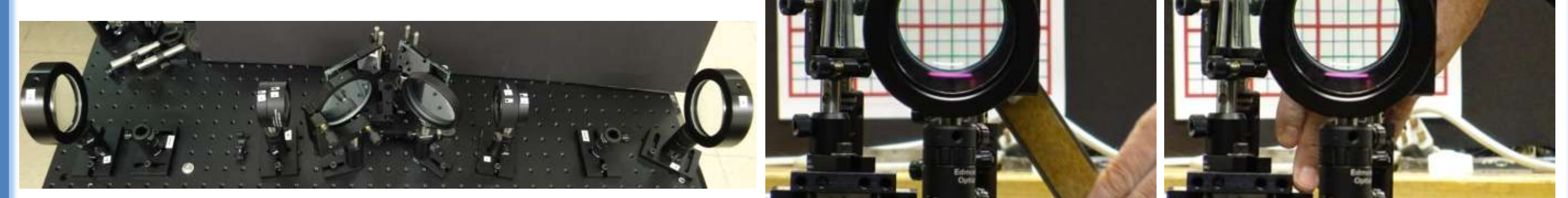
* (Need to check non-zero cloak volume separately.)

H. "Rochester Cloak"

- **4 lens cloak** that solves 'perfect' cloak ABCD matrix with cloakable volume.
- **Advantages**: Broadband for entire visible spectrum, 3D, isotropic, macroscopically scalable, and uses off-the-shelf optics.
- **Limitations**: Center not cloaked, $\sim 3^\circ$ field-of-view, edge effects, not full-field.
- **Alignment**: Sensitive. $\sim 1\%$ t_1, t_2, t_3 change causes magnification=1 to $\sim 50\%$.



• **Rochester Cloak version 2**: $\sim 2x$ field-of-view, 1.5x cloaking diameter, center-axis region cloaked.



I. Paraxial Full-Field Cloaking [2]

1) **Field propagation** (Huygens' integral in Fresnel (paraxial) approximation):

$$\tilde{E}_2 = i \frac{e^{-ik_0 L_0}}{B \lambda_0} \iint_{-\infty}^{\infty} \tilde{E}_1 \exp \left\{ -i \frac{\pi}{B \lambda_0} [A(x_1^2 + y_1^2) - 2(x_1 x_2 + y_1 y_2) + D(x_2^2 + y_2^2)] \right\} dx_1 dy_1$$

($L_0 = \sum_i n_i L_i$ = On-axis optical path length, (x, y) = Transverse space, λ_0, k_0 = Wavelength, wave vector ($2\pi/\lambda_0$) in vacuum, \tilde{E} = Complex field amplitude, (A, B, C, D) = Matrix coefficients)

2) **Full-field phase-matching**:

Add a thin, flat plate with index n_c and length L_c .

$$n_c(\lambda_0) = n(\lambda_0) + \frac{1}{L_c} \left(m \lambda_0 + \sum_i [n(\lambda_0) - n_i(\lambda_0)] L_i \right)$$

(m = Integer multiple for 2π phase-matching,



3) **Conclusions and Questions**:

- * Paraxial full-field cloaking = Isotropic.
- * **Broadband vs. Omnidirectionality**: \rightarrow Cannot have both for cloaking...? \rightarrow Can it be done with isotropic, ray optics?

J. References

- [1] J. Choi and J. Howell, *Opt. Express* **22**, 29465 (2014).
- [2] J. Choi and J. Howell, *Opt. Express* **23**, 15857 (2015).
- [3] U. Leonhardt, *Science* **312**, 1777 (2006).
- [4] J. Pendry, D. Schurig, and D. Smith, *Science* **312**, 1780 (2006).
- [5] D. Schurig et al., *Science* **314**, 977 (2006).
- [6] J. C. Howell, J. B. Howell, J. Choi, *Applied Optics* **53**, 1958 (2014).
- [7] H. Chen et al., *Nature Communications* **4**, 2652 (2013).

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