



HAJIM
SCHOOL OF ENGINEERING
& APPLIED SCIENCES
UNIVERSITY of ROCHESTER

THE INSTITUTE OF OPTICS



Advanced Labs

Beyond the First Year of College II

(University of Maryland, College Park, MD)

Multidirectional Invisibility with Rays of Light - A “Rochester Cloak”

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UNIVERSITY of ROCHESTER

Outline

1. Historical invisibility cloaking
 2. Scientific cloaking in 2006- “Transformation Optics”
 3. Initial ray optics cloaking- Unidirectional
 4. ‘Paraxial’ cloaking-
Multidirectional ray optics cloaking +
matching full-field/wave “phase”
- * We’ve tried to make this presentation self-contained and comprehensive; also, problem sets and demonstration guidelines are provided to help students as a lab course.



Invisibility in History and Fiction

- Greek “Cap of Invisibility” myths
 - Athena, Hermes and Perseus used it.
- Cloak of Invisibility
 - King Arthur, Jack the Giant Killer, Star Trek, Harry Potter, Lord of the Rings
- Chemicals
 - Invisible Man (H.G. Wells)



Invisibility in Magic Shows

- David Copperfield



- Science and Technology Museum
MadaTech



Define “Cloak” for Talk

- Not a wearable clothing, necessarily
- To “hide”
→ What we’ll use



Active Camera Cloaks

- Tachi Lab, Keio University, Japan
 - Original in 2003 ([Demo](#))
- Mercedes-Benz campaign in 2012 ([Mercedes-Benz link](#))
- Land Rover “Transparent Hood” (2014)



A New Beginning for Scientific Cloaking (2006)

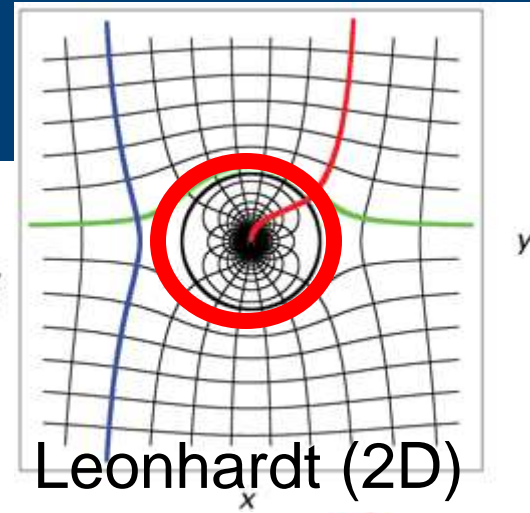
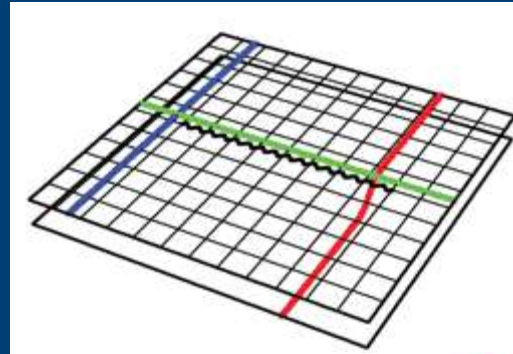
“TRANSFORMATION OPTICS”



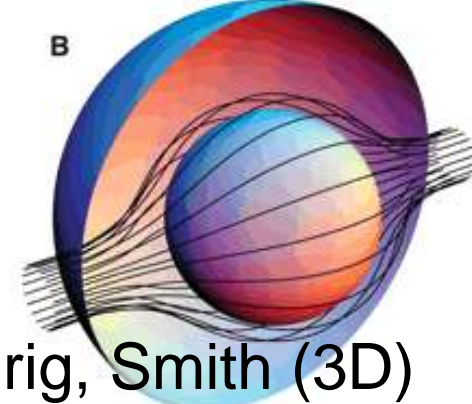
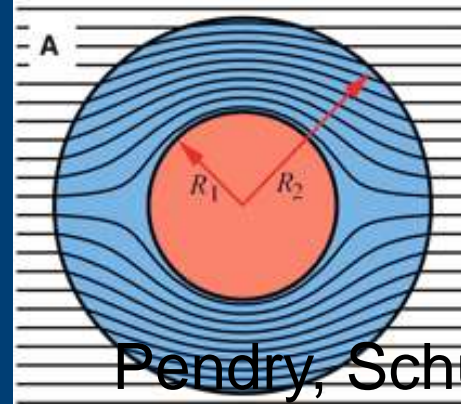
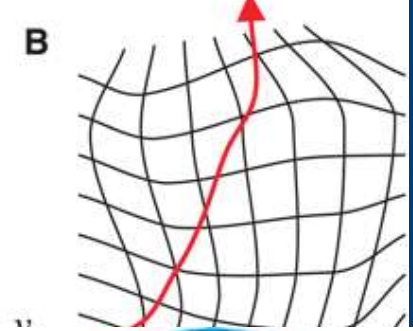
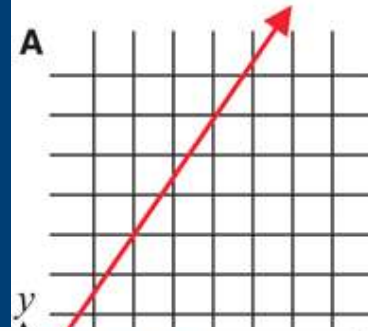
Transformation Optics

1. Create virtual space with **region** that light does not enter.
2. Map this to physical space through coordinate transformation.
3. Build physical space with artificial materials ('metamaterials') only.

→ In 2006, 2 research groups (*Science*)



Leonhardt (2D)

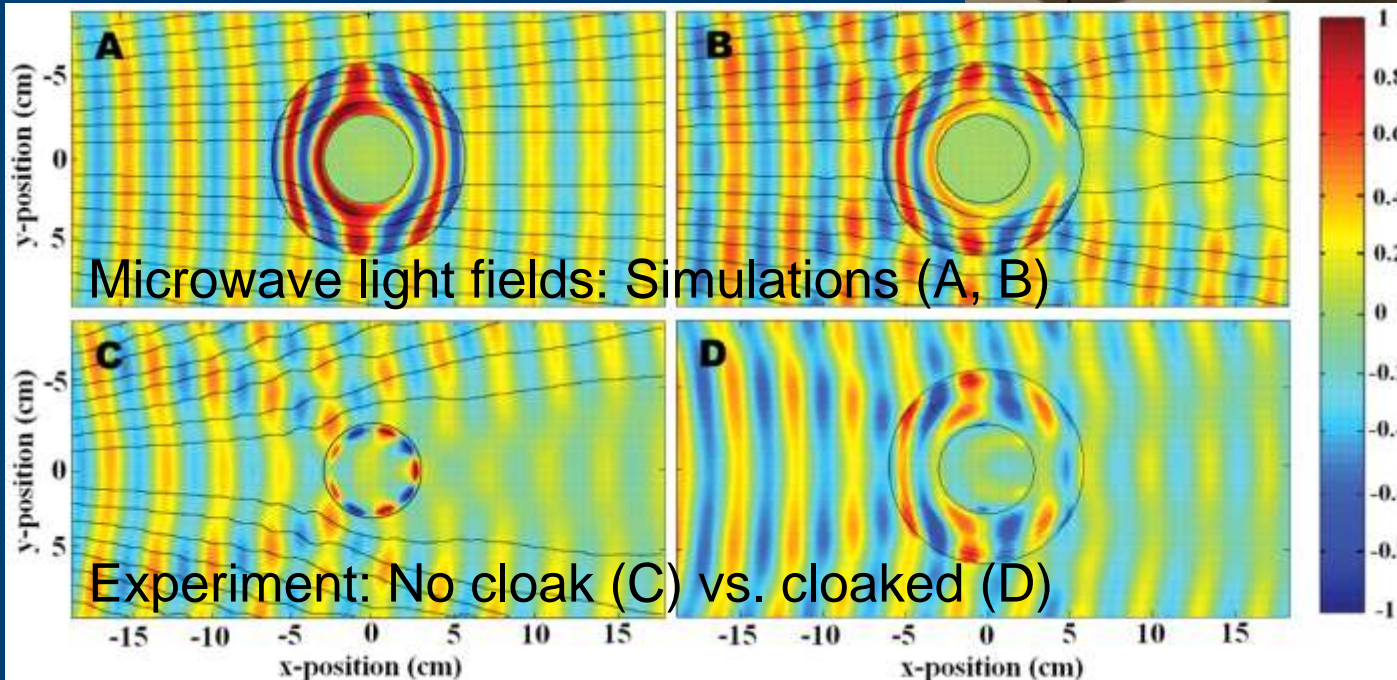
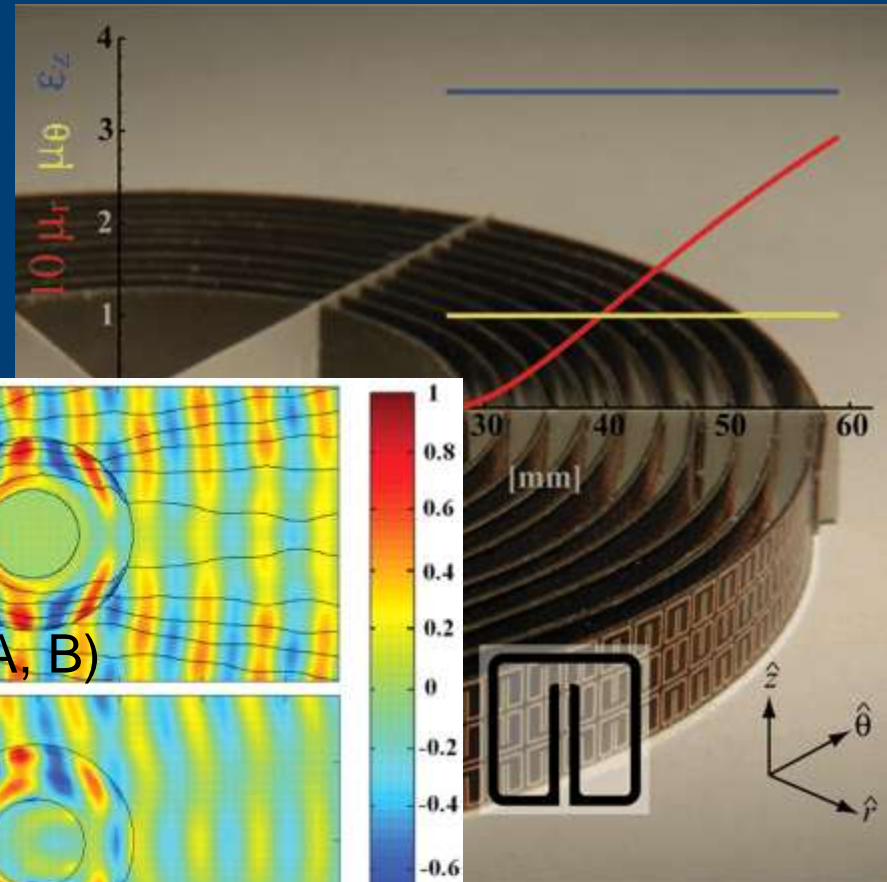


Pendry, Schurig, Smith (3D)



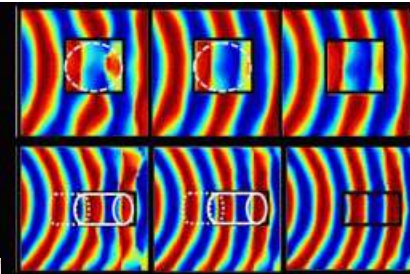
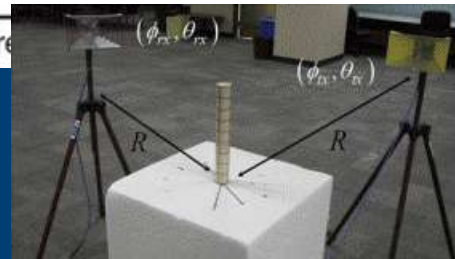
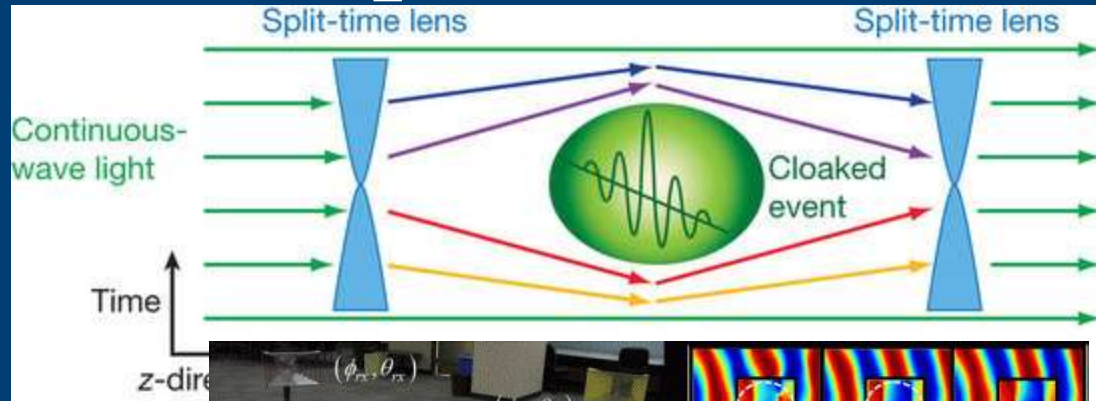
Microwave 2D Cloak (2006)

- First demonstration using Transformation Optics (Schurig et al.)
- For 2D, microwave using “split-ring resonators” (metamaterial)

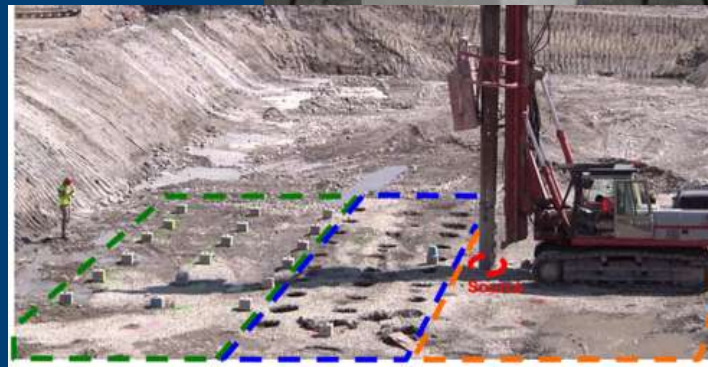


Transformation Optics (1)

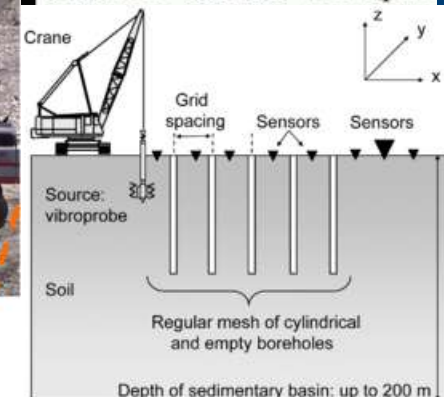
- Revolutionary for material design applications and cloaking.
- Omnidirectional
- Full field cloaking for entire light wave (phase + amplitude)
- Examples:
 - Time cloaking
 - Thin, radio wave cancelling cloak
 - Seismic cloaking



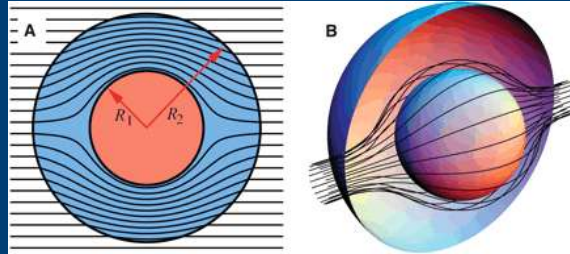
Unlocked Cloaked Free Space



Sensitive three components velocimeters (green grid) Five meters deep 320 mm holes Source :
 - Frequency : 50 Hz
 - Horizontal displacement : 14 mm



Why Ray Optics Cloaking?



<u>'Ideal' Cloak Properties</u>	Transformation Optics	Initial Ray Optics Cloaking
Broadband	Difficult	Excellent
Visible spectrum		
Isotropic		
Macroscopic scalability		
3D	Some challenges	~No (1 or discrete freq.)
Full-field (phase+amplitude)	Excellent	
Omnidirectional		1 or discrete directions



Unidirectional

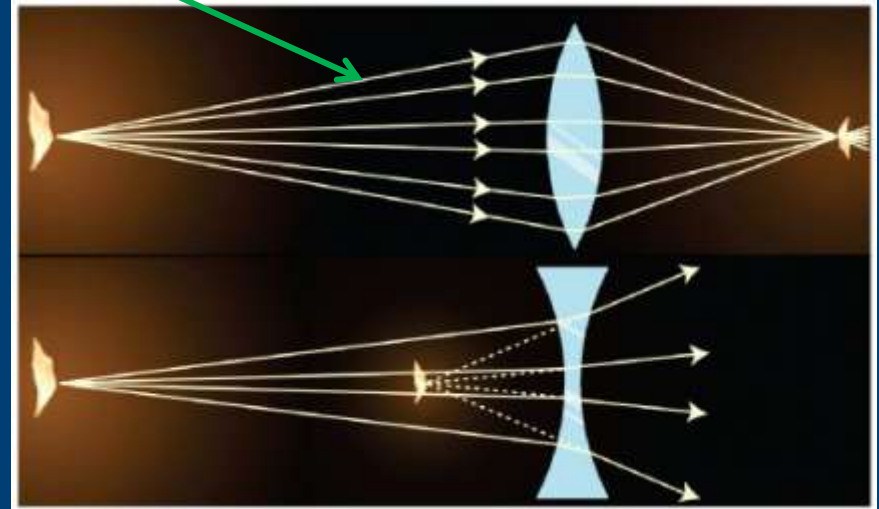
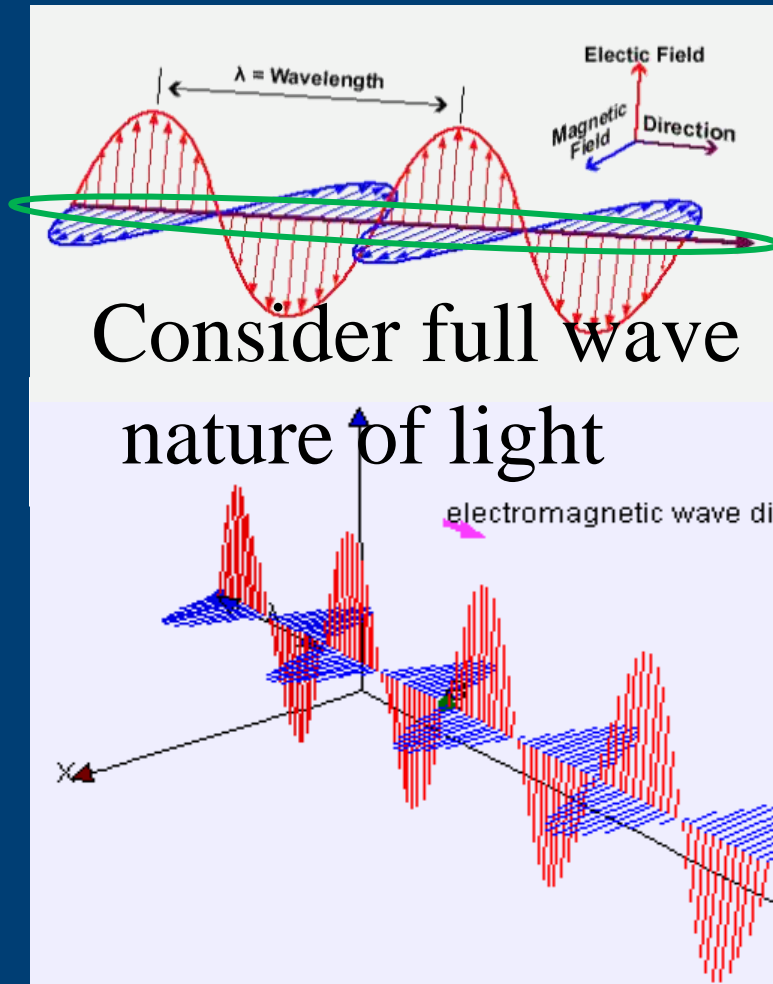
RAY OPTICS CLOAKING



Full Field Optics

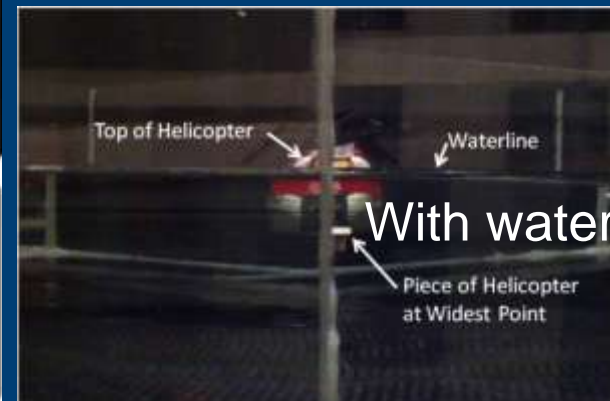
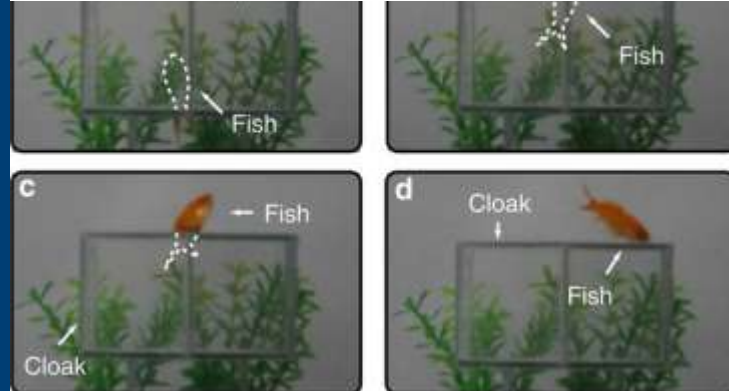
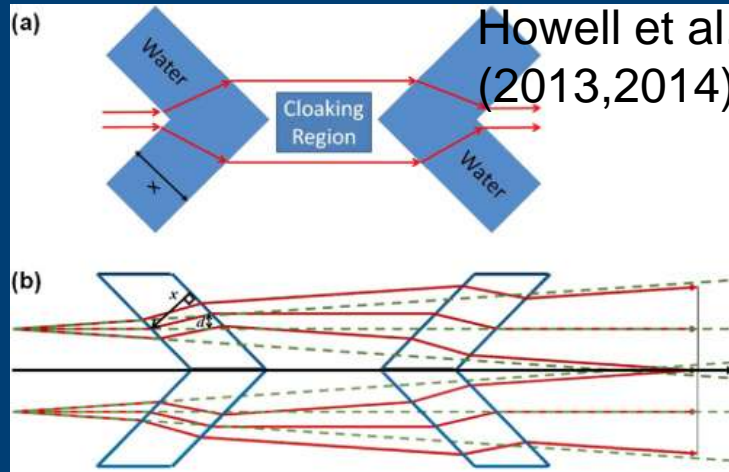
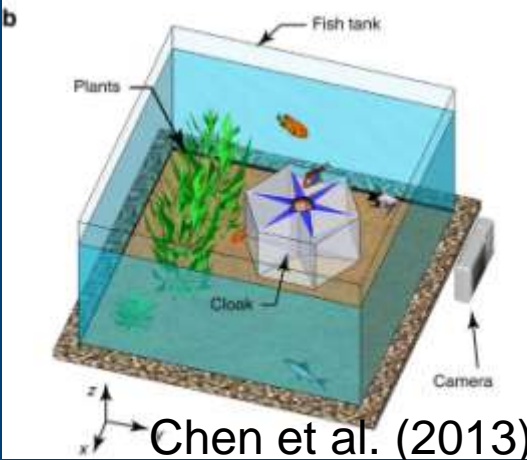
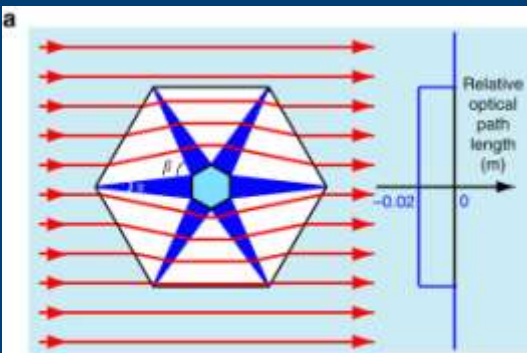
Ray Optics

- Only consider direction and power
- Easier



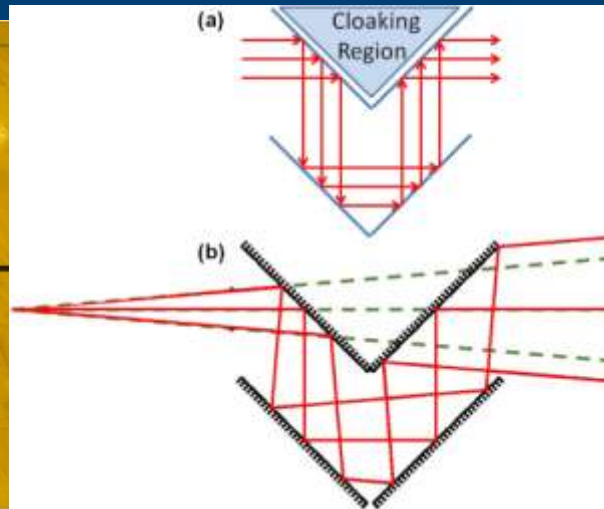
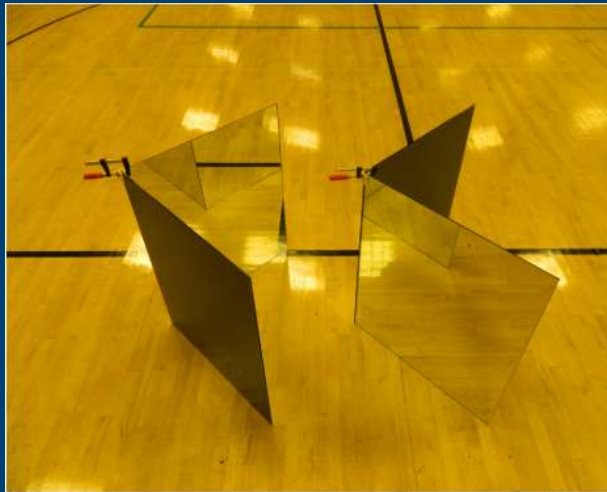
Ray Optics Cloaking

- Macroscopic, visible light cloaks
- Unidirectional, or discretely multidirectional
- Other directions: Background shift, cloak revealed



UR Ray Optics Mirror Cloak (2013)

- University of Rochester (UR)- Prof. John Howell and sons (2013 in arXiv)
- Magnification not 1, unidirectional



[J. C. Howell, J. B. Howell, and J. S. Choi, Applied Optics 53, 1958 \(2014\).](#)



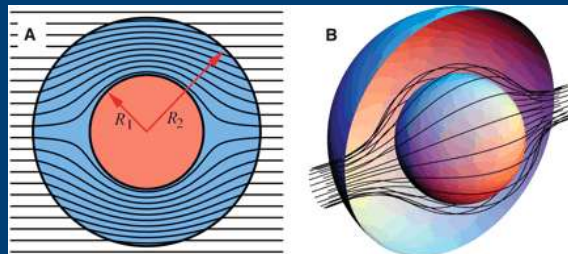


Opt. Express 22, 29465-29478 (2014)

PARAXIAL RAY OPTICS CLOAKING



Why Paraxial Ray Optics Cloaking?



<u>'Ideal' Cloak Properties</u>	Transformation Optics	Paraxial Ray Optics Cloaking
Broadband	Difficult	Excellent
Visible spectrum		
Isotropic		
Macroscopic scalability		
3D	Some challenges	~No (1 or discrete freq.)
Full-field (phase+amplitude)	Excellent	
Omnidirectional		Continuous multidirections



Geometric Optics and ABCD Matrices

TUTORIAL



Geometric Optics Tutorial:

Snell's Law

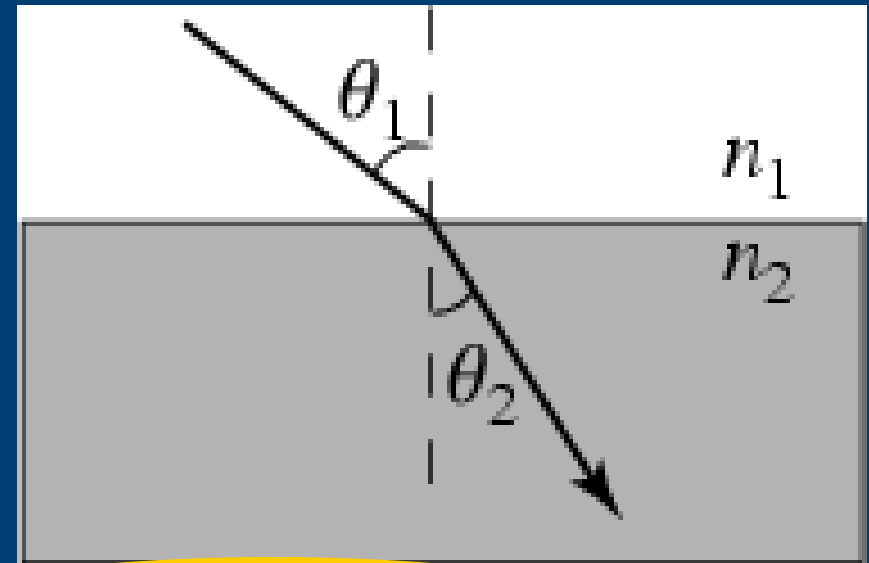
$$n_1 \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$$

(n = index of refraction)

- Can derive from Fermat's Principle.
- Use to “trace” light rays.

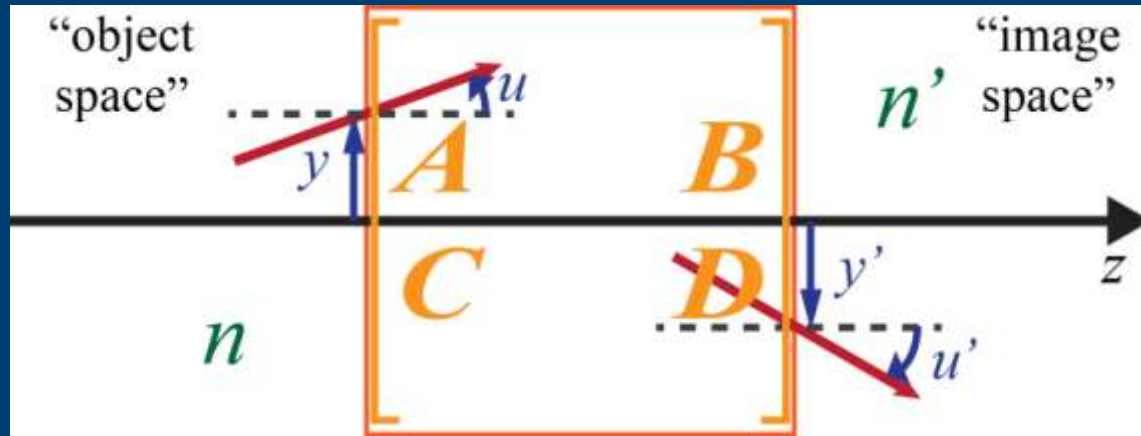
- 1st order expansion $\rightarrow \sin \theta \approx \theta \approx \tan \theta \equiv u$

(‘Paraxial’ approximation = small-angle approximation)



Geometric Optics Tutorial:

ABCD Matrix & Light Rays



- Combined optical system = Modeled by ABCD matrix.

- Light rays:

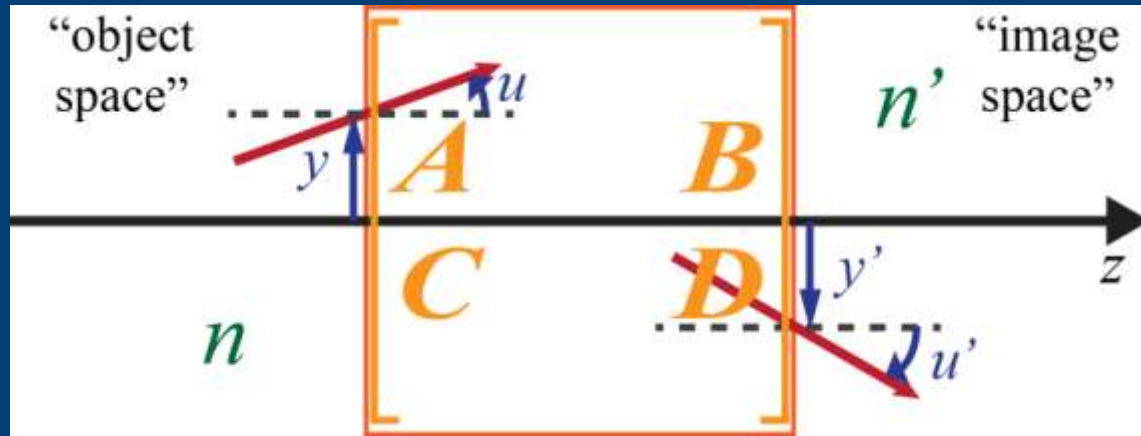
- y, y' = Position of input, output (') rays.
- u, u' = Direction of input, output(') rays.
- n, n' = Refractive index of input, output(') spaces.

- ABCD matrix: Given input ray, determines output ray, through matrix multiplication.

$$\begin{bmatrix} y' \\ n' u' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} y \\ n u \end{bmatrix}$$



Geometric Optics Tutorial: ABCD Matrix Assumptions



■ Assumptions:

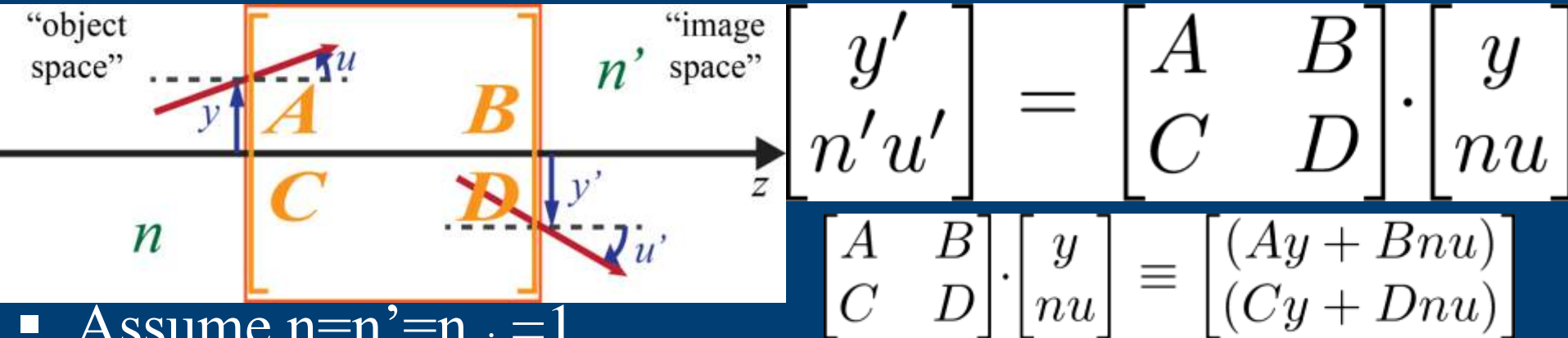
- Rotational symmetry: Everything is the same when rotated about center axis (z).
- Paraxial = small-angle. Question: How large is “paraxial?”
Hint: Consider when $(\sin \theta) = (\tan \theta)$ holds.
- Light travels from left to right (Rochester convention).

- Units: $[y] = \text{length}$, $[u] = [\tan \theta] = \text{unitless angle}$.



Geometric Optics Tutorial:

Problem Set: Basic ABCD Matrices



- Assume $n=n'=n_{\text{air}}=1$.

1) What is ABCD matrix of air of length L ?

Hint: Rays continue same path. Draw an input ray, and intuit what output ray should be like. Then find matrix to make it so.

2) What is ABCD matrix of thin lens of focal length f ?

(Challenge problem)

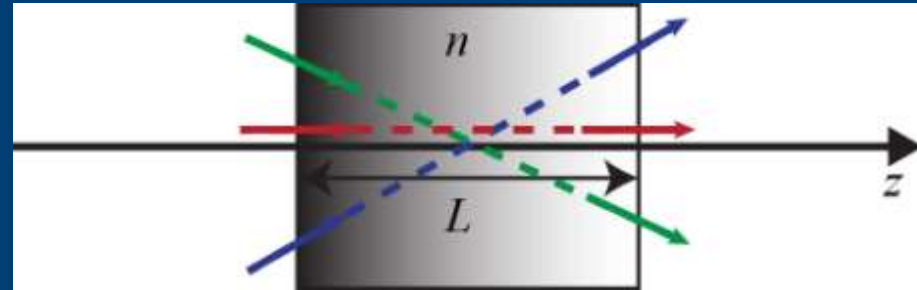
Hint: A collimated input beam will focus at distance f . Can assume that the lens is infinitely thin.



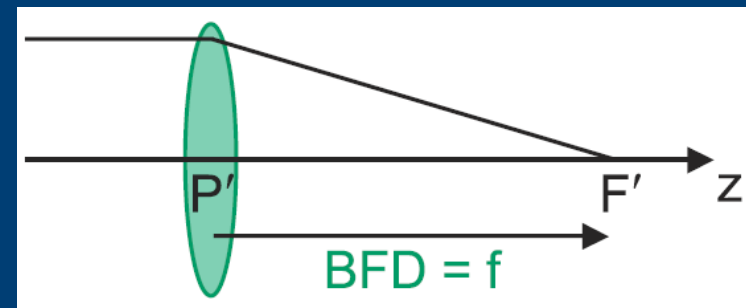
Geometric Optics Tutorial:

Basic ABCD Matrices

- For general n, n' .
- ABCD matrix for space of length L with index n :
 - Draw rays.
- ABCD matrix for thin lens of focal length f , in index n space*:
(* When $n \neq 1$, care required for “ f ”)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{translation}} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{thin lens}} = \begin{bmatrix} 1 & 0 \\ -n/f & 1 \end{bmatrix}$$



Geometric Optics Tutorial: Matrix Multiplication

1) ABCD *matrix* multiplying a $[y, nu]$ *vector*:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} y \\ nu \end{bmatrix} \equiv \begin{bmatrix} (Ay + Bnu) \\ (Cy + Dnu) \end{bmatrix}$$

2) Matrix multiplying a matrix:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \equiv \begin{bmatrix} (A_1A_2 + B_1C_2) & (A_1B_2 + B_1D_2) \\ (C_1A_2 + D_1C_2) & (C_1B_2 + D_1D_2) \end{bmatrix}$$

3) Multiplication of 3 matrices:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \\ \left(\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \right) \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \left(\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \right)$$



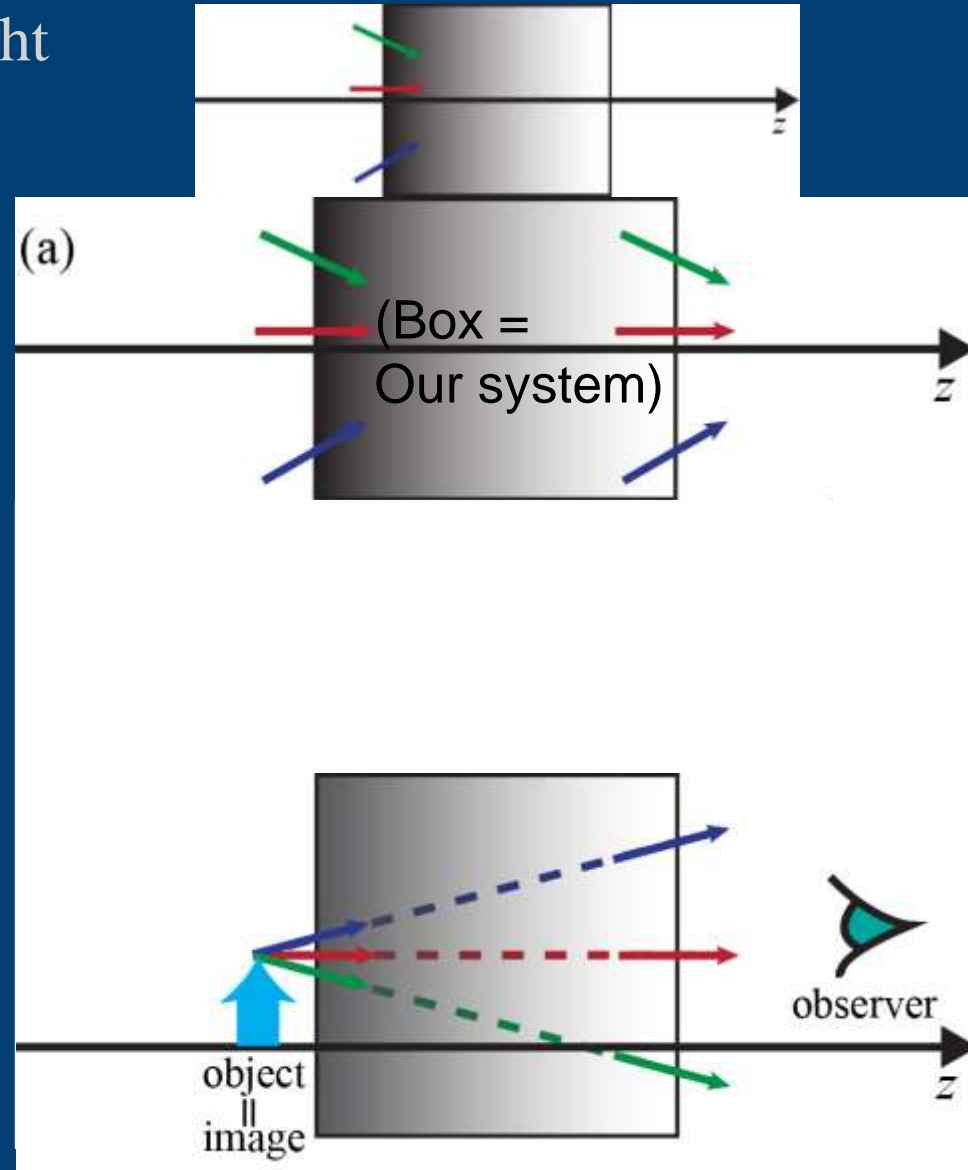
Back to...

PARAXIAL RAY OPTICS CLOAKING



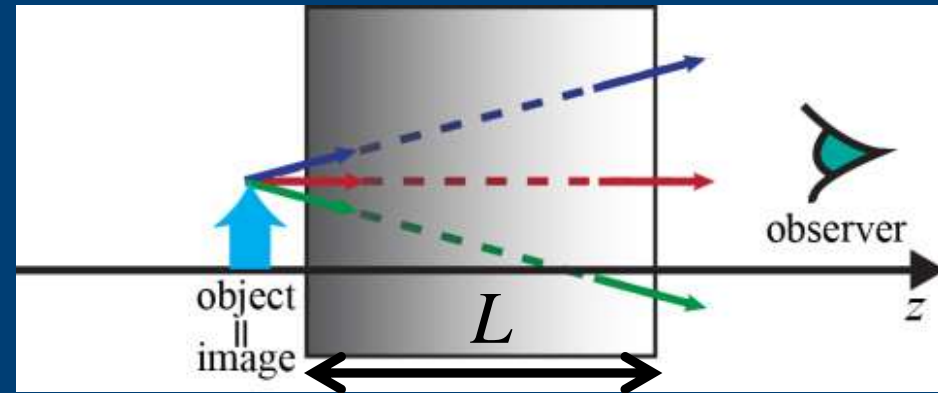
Fun Problem: What is a Cloak?

- Draw a few input and output light rays for a cloaking box.
- Attempt 1:
Don't do anything and keep same rays.
- Result? Image \neq Object
- Attempt 2: Don't do anything, really...
- Result?
System = Empty space
→ Image = Object **YES!**



Cloaking: Paraxial Geometric Optics

- Use ‘paraxial’ formalism (small-angle $\sim 30^\circ$ or less).
- Assume $n=n'=n_{\text{air}}=n_{\text{free space}}=1$.
- Perfect Cloak:
 1. System = Empty space of same length (L)
 2. Non-zero volume hidden
- ABCD Matrix = ? (Think...)



‘Translation’ Matrix

→ Object + device = empty space.

Note: Geometric Optics formalism is inherently 3D and multidirectional.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{PerfectCloak}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$



Paraxial Cloaking Problem Set 1

- Use rotationally symmetric, thin lenses only.
- Problem:** What conditions are required to satisfy cloaking matrix, for

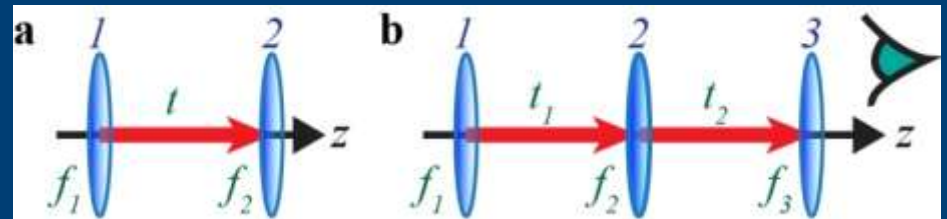
- 1 lens with focal length f_1 ?
- 2 lenses?
- 3 lenses (Challenge problem)?

* Hint: Multiply matrices first.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{PerfectCloak}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

- Answers from students:

- 1 lens:
- 2 lenses:
- 3 lenses:



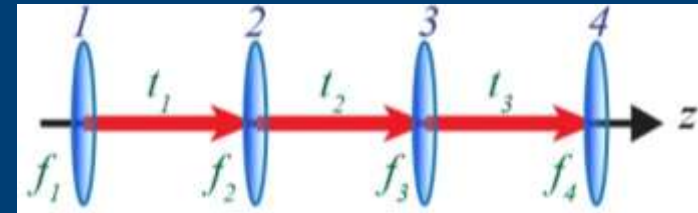
$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \equiv \begin{bmatrix} (A_1 A_2 + B_1 C_2) & (A_1 B_2 + B_1 D_2) \\ (C_1 A_2 + D_1 C_2) & (C_1 B_2 + D_1 D_2) \end{bmatrix}$$

$$\left(\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \right) \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \left(\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \right)$$



Paraxial Cloaking Problem Set 2

- Problem:** Show that a left-right symmetric 4 lens setup with focal lengths $f_1 = f_4$, $f_2 = f_3$, and distances $t_1 = t_3$, satisfies the “Perfect Cloak” ABCD matrix, *if*:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{PerfectCloak}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

a) $t_1 = f_1 + f_2$,

b) $t_2 = 2 f_2 (f_1 + f_2) / (f_1 - f_2)$.

- Hint: First obtain total ABCD matrix of the combined 4 lens system, by multiplying 7 ABCD matrices for the 7 “optical elements.”

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \equiv \begin{bmatrix} (A_1 A_2 + B_1 C_2) & (A_1 B_2 + B_1 D_2) \\ (C_1 A_2 + D_1 C_2) & (C_1 B_2 + D_1 D_2) \end{bmatrix}$$

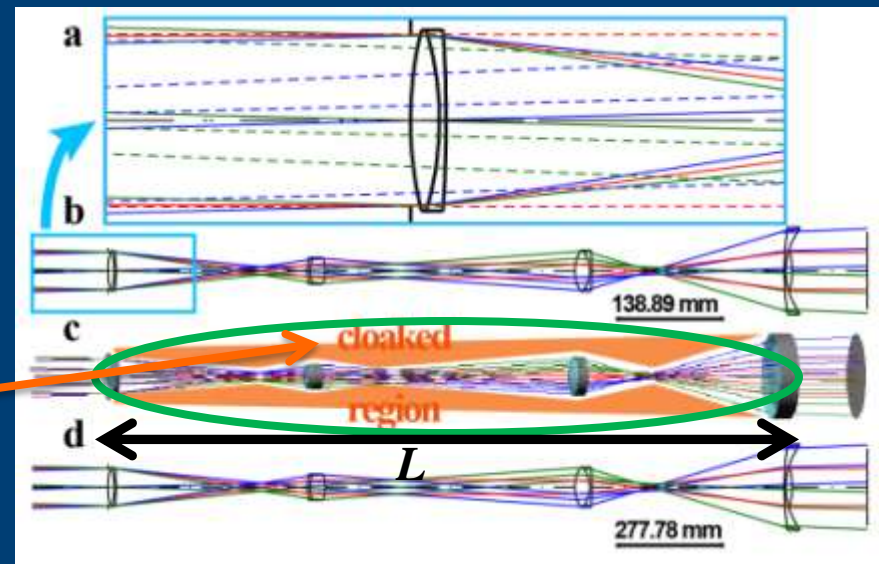
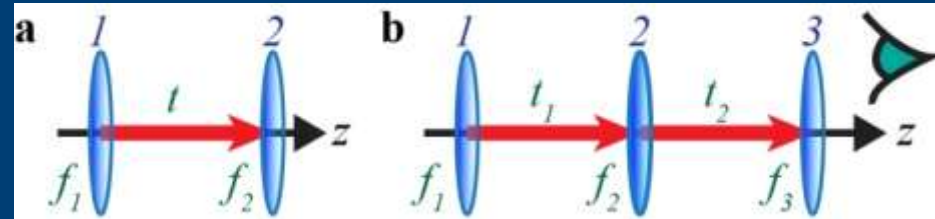
$$\left(\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \right) \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \left(\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \right)$$



Paraxial Cloaking Design

- Try to find simplest design that satisfies:
- Use rotationally symmetric, thin lenses.
- 1-2 lenses: No optical power, so no cloakable space.
- 3 lenses: Asymptotically can approach 'perfect' cloak.
- At least 4 lenses required to build 'perfect' cloak:
 - System = Empty space of same length.
 - Non-zero volume to hide an object.

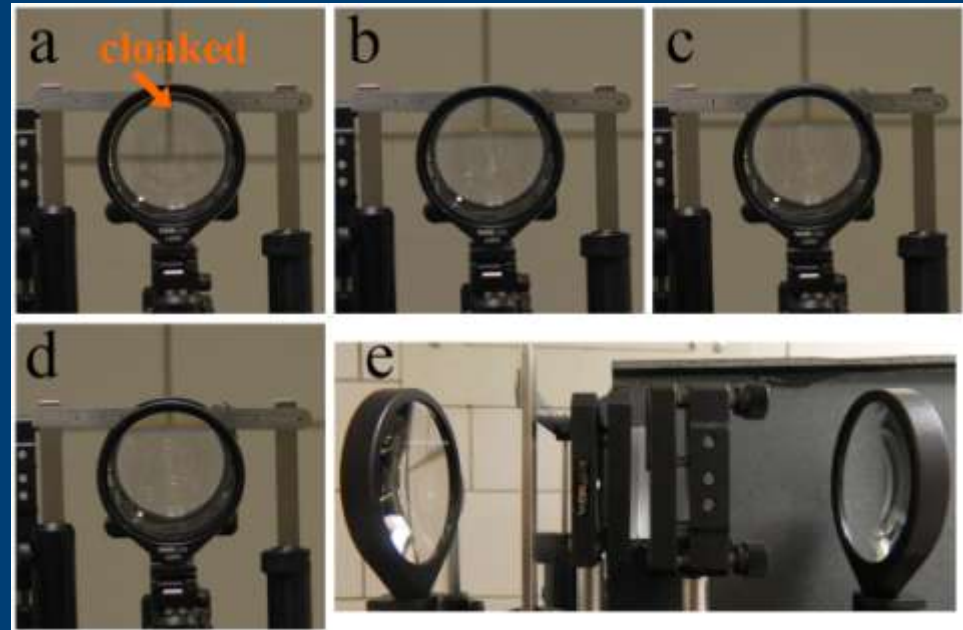
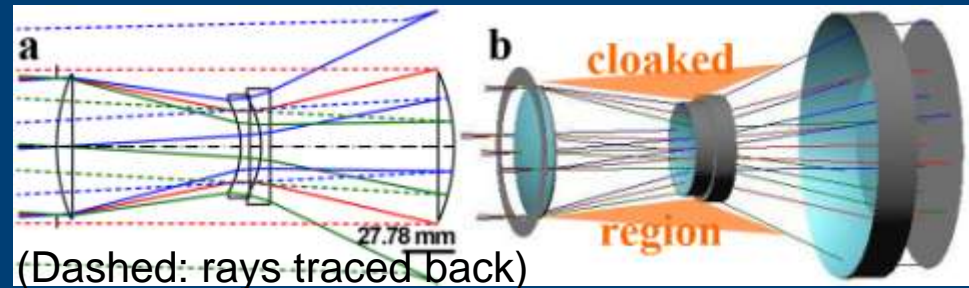
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{PerfectCloak}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$



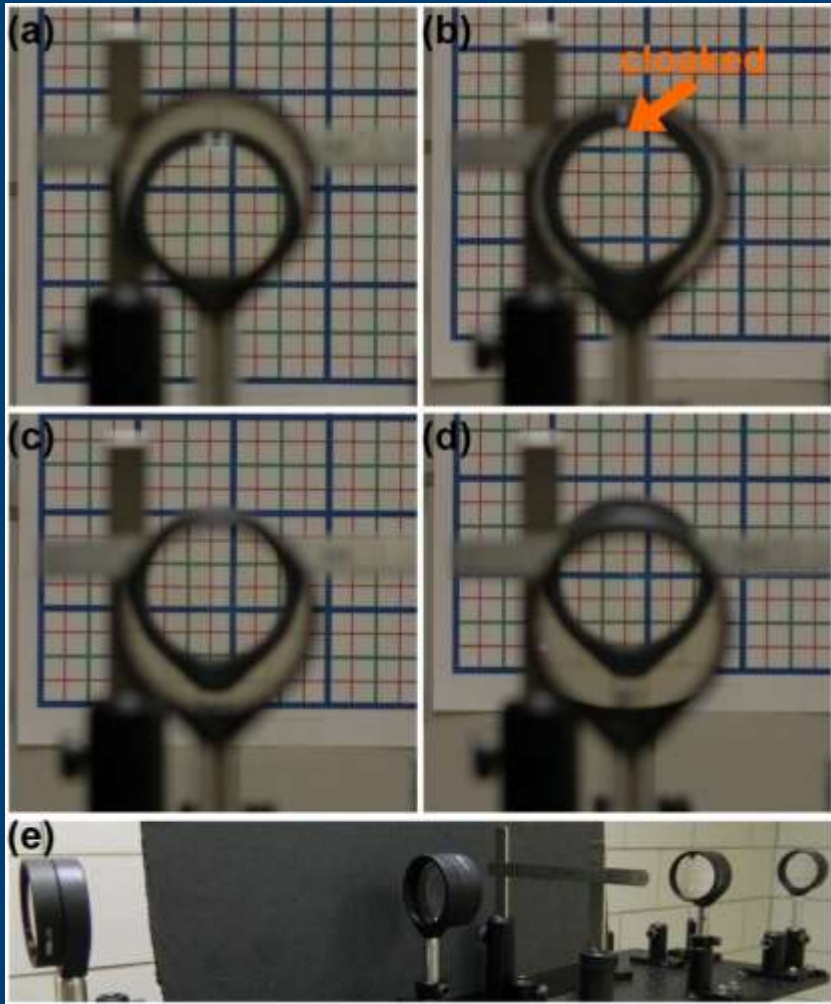
Symmetric 3 Lens Cloak

Though not quite ‘perfect’ cloak, still works decently.

1. Magnification not 1, nor afocal.
2. Background image close to matching.
3. Cloaking works for continuous range of directions.



4 Lens “Rochester Cloak” Results



1. Background image matches (lenses = empty space).
→ Magnification = 1, afocal (no net focusing power)
2. Cloaking works for continuous range of directions.
3. Edge effects (paraxial nature), center axis must not be blocked.

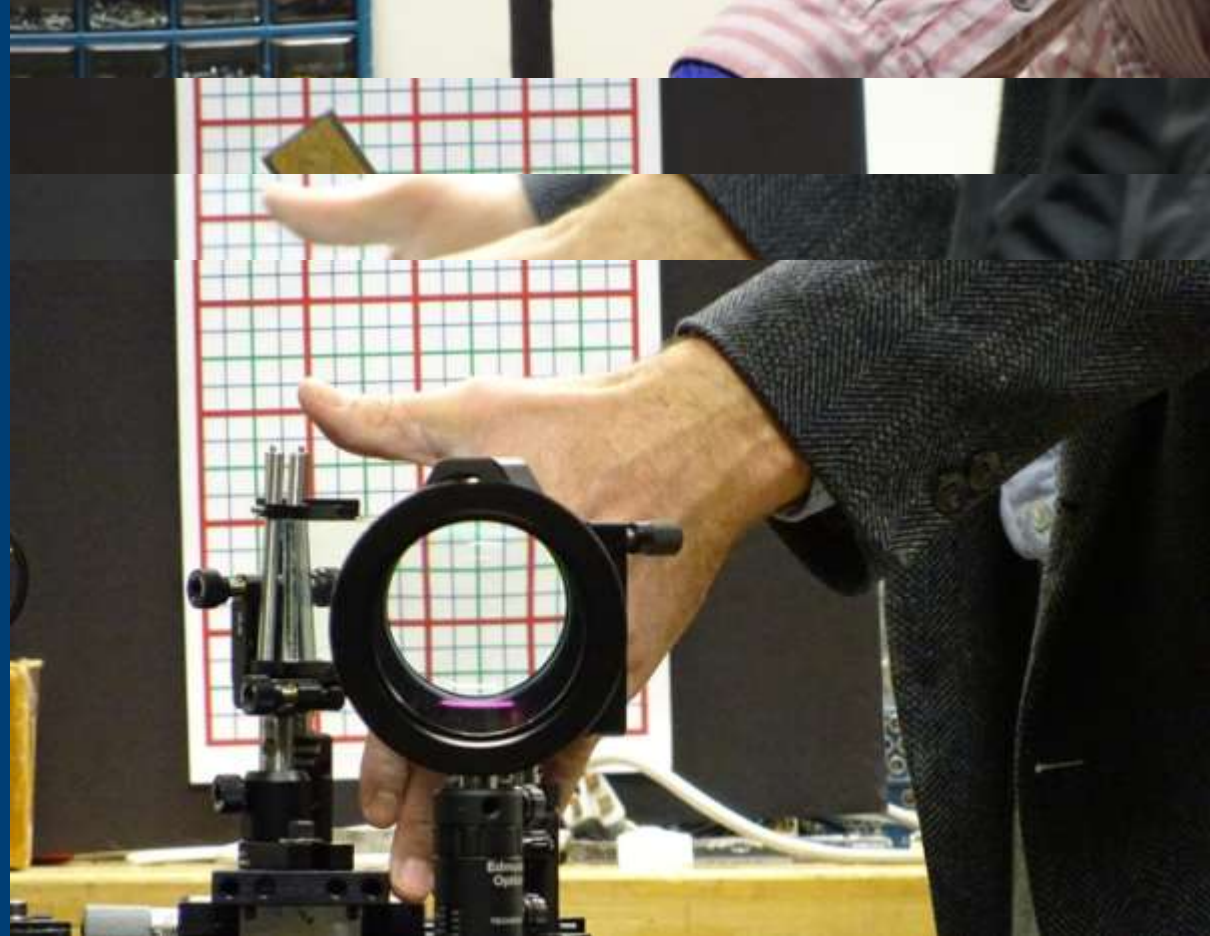
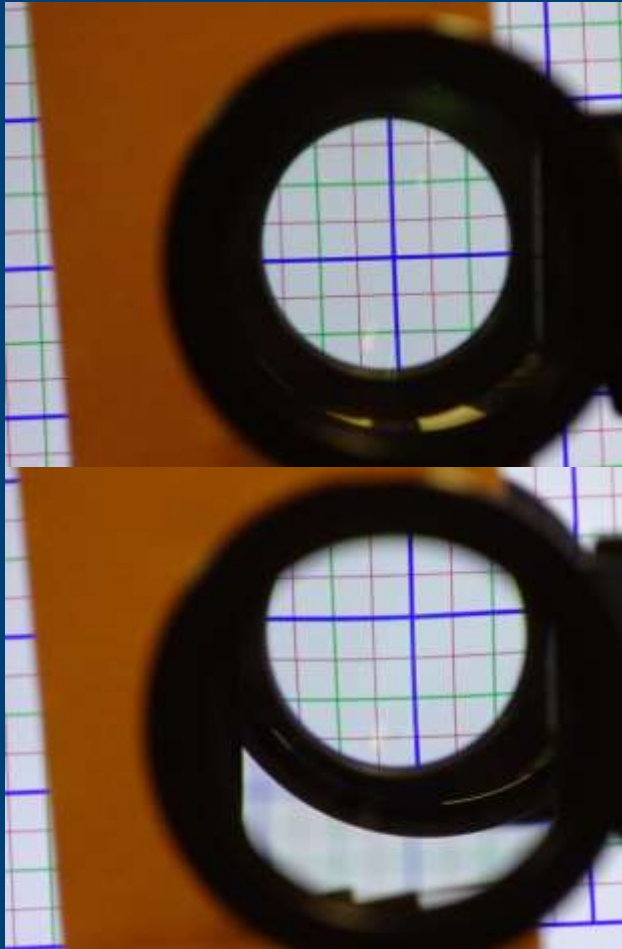
(*Optics Express*, Vol. 22, pp. 29465-29478, 2014)



Rochester Cloak 2

Edmund Optics 3" achromats:

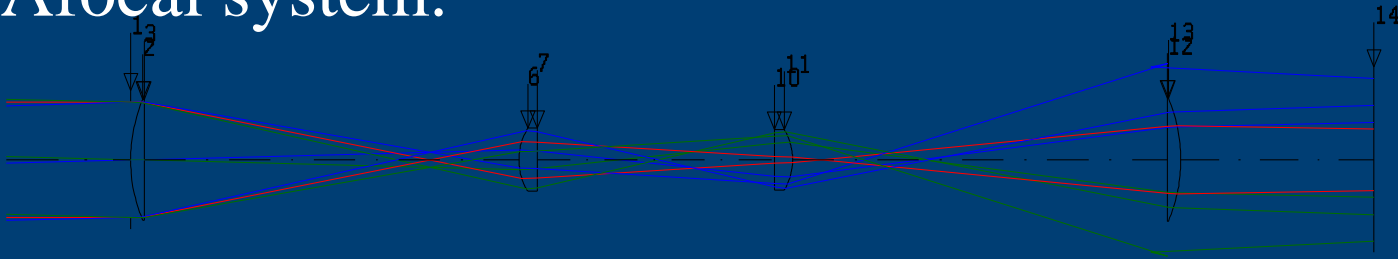
- ~2x field-of-view, 1.5x cloaking diameter (compared to original).
- Center-axis region cloaked as well.



Optical Engineering Work?

- Design Requirements:

- Magnification of 1.
 - Afocal system.

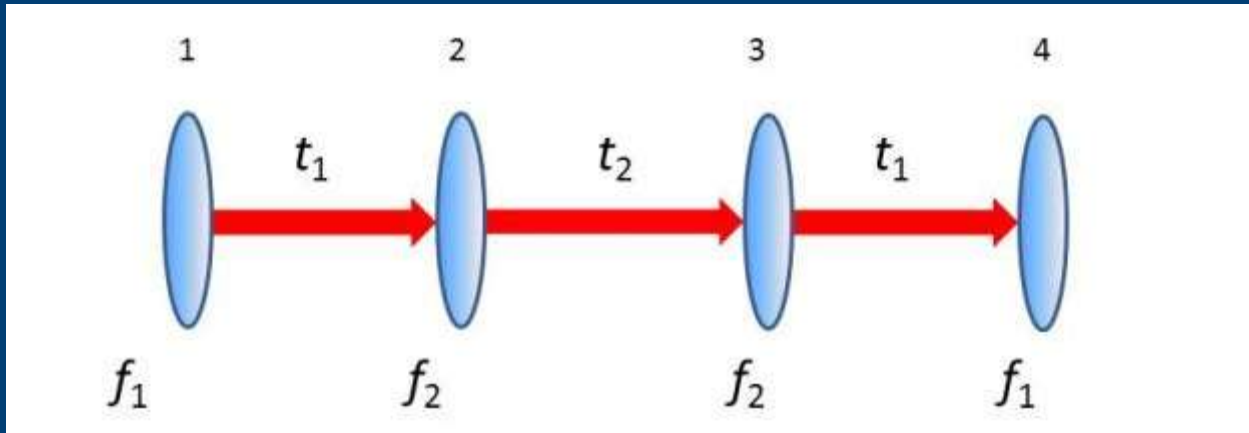


- Replicate ambient medium (“NonLens” solution).
- Explore >4 lenses and other optical elements.

P. P. Clark and C. Londono, “1990 International Lens Design Conference lens design problems: the design of a NonLens,” in “1990 Intl Lens Design Conf,” (Proc. SPIE, 1991), 1354, pp. 555–569.

113.64 MM

Alignment



- Very sensitive to distances between lenses:
~1% change in t_1 , t_2 , t_3 can change magnification = 1 to ~50% instead; 1mm counts.
- Tips:
 - Account for lens surface location on mount .
 - Use collimated input beam and check for collimation after lenses 1 & 2, lenses 3 & 4 pairs.
 - Magnification should be 1.
 - t_2 controls the image for multidirectional viewing angles.

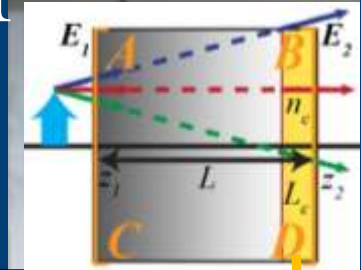
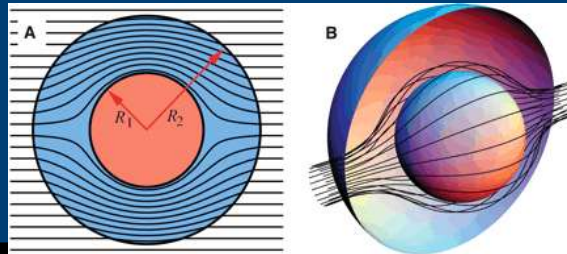


Opt. Express, 23, 15857 (2015)

PARAXIAL FULL-FIELD CLOAKING



Experimental Cloaking Comparison



<u>'Ideal' Cloak Properties</u>	Transformation Optics	Paraxial Full-field Cloaking
Broadband	Difficult	Excellent
Visible spectrum		
Isotropic		Some challenges
Macroscopic scalability		Excellent
3D	Excellent	Broadband (theory)
Full-field (phase+amplitude)	Excellent	Continuous multidirections
Omnidirectional	Excellent	Continuous multidirections

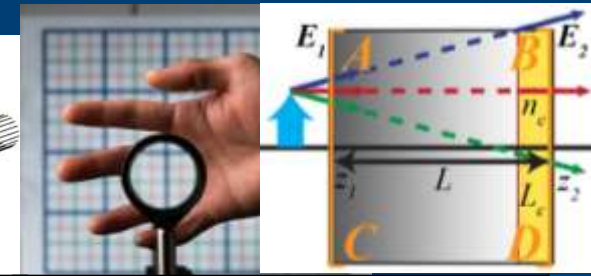
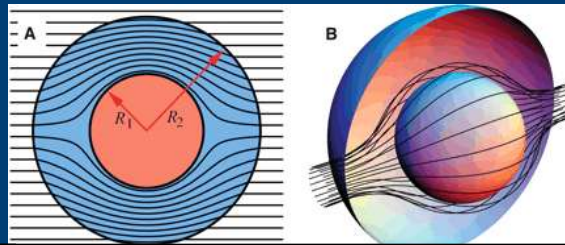


Combine...

PARAXIAL CLOAKING



Cloaking Comparison Redux



<u>'Ideal' Cloak Properties</u>	Transformation Optics	Paraxial Cloaking
Broadband	Difficult	Excellent
Visible spectrum		
Isotropic		
Macroscopic scalability	Some challenges	<u>Broadband</u> (theory)
3D	Excellent	Continuous multidirection
Full-field (phase+amplitude)		
Omnidirectional		

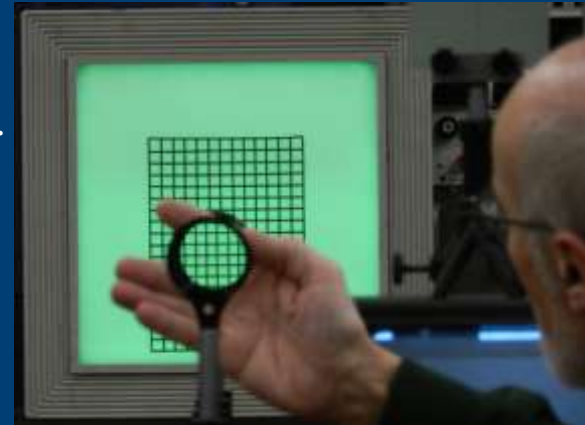
- Broadband vs. Omnidirectionality: Cannot achieve all?!
- Anisotropy still not required for paraxial cloaking.
- Isotropic, broadband, omnidirectional cloak possible for ray optics?



Other References

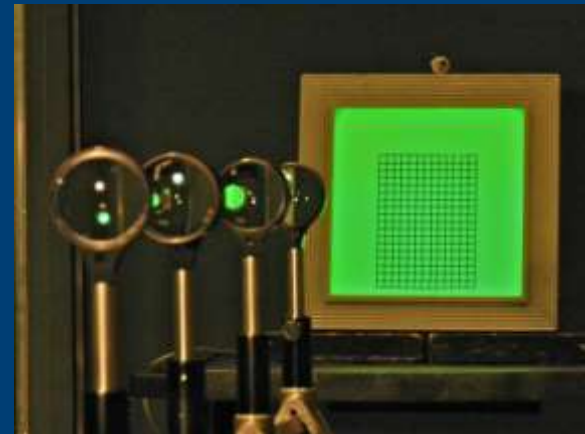
1) Thick lens distance calculations for “Rochester Cloak”

- a) **Notes:** “ t_1 ,” “ t_2 ” are distances between the two closest surfaces (not the lens centers). These distances are what we used, and were calculated using thick lens ABCD matrices only. These will change if lenses are different than those from 2014-15.
- b) Edmund Optics achromatic doublets:
“ t_1 ” = 248.9 mm, “ t_2 ” = 333.9 mm.
Total length (L) = 928.5 mm = $(333.9 + (248.9 + 23.94 + 24.5) * 2)$.
Lenses 1 & 4 (# 45-417): $f=200$ mm, $\text{Ø}3$ ”,
Lenses 2 & 3 (# 49-291): $f=75$ mm, $\text{Ø}2$ ”.
- c) Thorlabs achromatic doublets:
“ t_1 ” = 255.49 mm, “ t_2 ” = 332.17 mm.
Total length (L) = 910 mm = $(332.17 + (255.49 + 10.5 + 23) * 2)$.
Lenses 1 & 4 (AC508-200-A-ML): $f=200$ mm, $\text{Ø}2$ ”,
Lenses 2 & 3 (AC508-075-A-ML): $f=75$ mm, $\text{Ø}2$ ”.



2) Harvard lab: “Paraxial Ray Optics Cloaking” (<http://sciencedemonstrations.fas.harvard.edu/>)

- Additional Notes:
Works for any object/image distance.
Better to hide object between 1st and 2nd lenses,
or 3rd and 4th lenses, not 2nd and 3rd lenses (center).

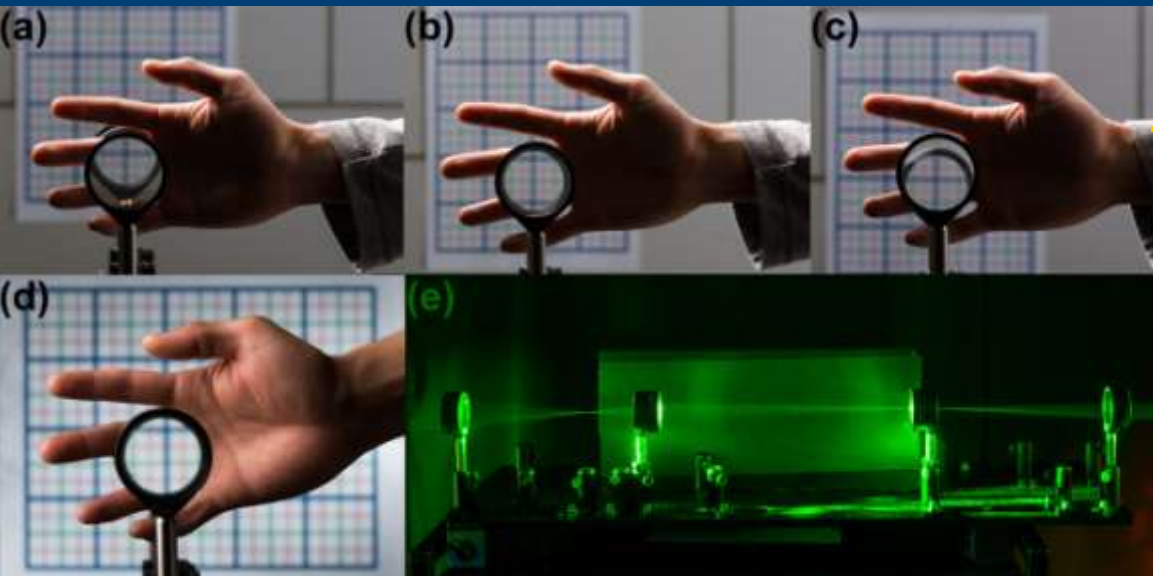


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- AAPT, BFY II committee
- Edmund Optics



(Photos by J. Adam Fenster / University of Rochester)



Demonstration

Thank you!

Questions?

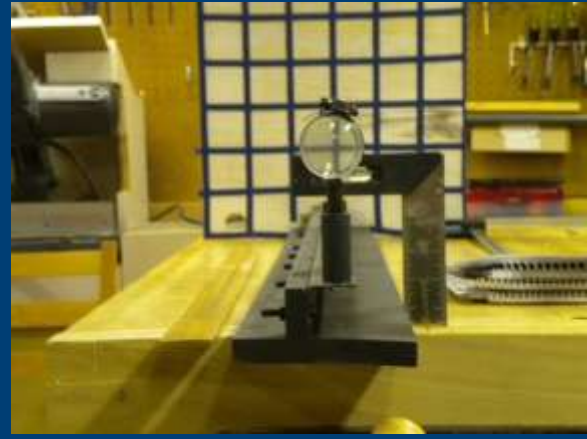
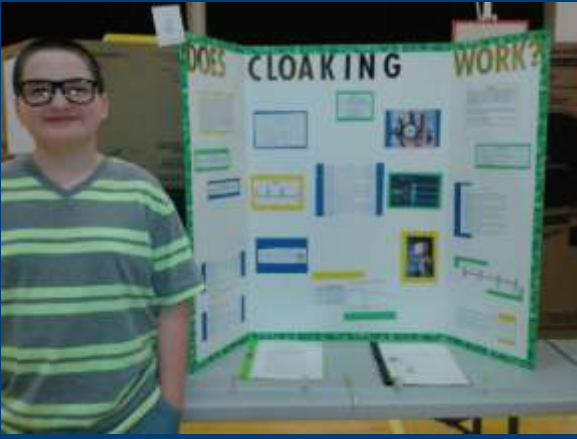
joseph.choi@rochester.edu

(*Optics Express*, Vol. 22, pp. 29465-29478, 2014)

Joshua G. (7th grade)

Joel & Linda D.

Josh O. (8th grade)



Demonstration Guide 1- with camera

1. Place background object, such as a grid sheet, behind cloaking device.
2. Use camera on a tripod about 2-3m in front of cloaking device, and zoom in fully to view through all the lenses. (The camera being farther away produces better image since the cloak was designed for small field-of-view/angles.)
3. Project camera onto screen for all to view.
4. Move camera horizontally or vertically. Background image seen through all lenses should match background object outside of the lenses. (See Fig. 5 of paper)
5. Place fingers or an object in between 1st and 2nd lenses, but avoid center axis. These should disappear, i.e. be cloaked. Caution them to not touch the lenses.
6. Explore other regions that can cloak the objects. (See Fig. 4(c) of paper) While object is cloaked, move camera to observe multidirectional cloaking effect.
7. OPTIONAL 1 (Be careful): Misalign only one of the lenses by moving it up/down, turning /rotating it. This should show how alignment is important. Have student correctly re-align by matching background image to object.
8. OPTIONAL 2 (Be very careful): Misalign one of the lenses in distance to other lenses. Other effects of misalignment will be seen, and students can use ruler to reposition lens. CAUTION: This requires precise and careful alignment, and may be difficult to get back to optimal alignment, but helps students develop essential and practical alignment skills for a real lab.
9. OPTIONAL 3 (Advanced): Misalign multiple lenses in multiple ways. Rebuild cloak.

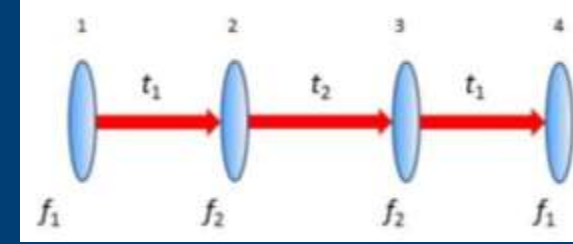


Demonstration Guide 2- without camera

1. Place background object, such as a grid sheet, behind cloaking device.
2. Have students stand about 2-3m in front of cloaking device, and view through all the lenses, one at a time. (The farther away an observer is, the better the image, since the cloak was designed for small field-of-view/angles.)
3. Have the students move their heads horizontally or vertically. Background image seen through all lenses should match background object outside of the lenses. (See Fig. 5 of paper)
4. One student can place fingers or objects in between 1st and 2nd lenses, but avoid center axis. These should disappear, i.e. be cloaked. Caution them to not touch the lenses. The observer will have to tell the student when the fingers or objects disappear from view.
5. Explore other regions that can cloak the objects. (See Fig. 4(c) of paper). While object is cloaked, move head to see the multi-directional cloaking effect.
6. OPTIONAL 1 (Be careful): Misalign only one of the lenses by moving it up/down, turning /rotating it. This should show how alignment is important. Have student correctly re-align by matching background image to object. This will require collaboration- The person viewing must give feedback to the student aligning.
7. OPTIONAL 2 (Be very careful): Misalign one of the lenses in distance to other lenses. Other effects of misalignment will be seen, and students can use ruler to reposition lens. CAUTION: This requires precise and careful alignment, and may be difficult to get back to optimal alignment, but helps students develop essential and practical alignment skills for a real lab.
8. OPTIONAL 3 (Advanced): Misalign multiple lenses in multiple ways. Rebuild cloak.



Example Lab Suggestions



Goal: Build a 4 lens cloaking device from the beginning.

1. Start with 2 lenses with focal length f_1 , and 2 others with f_2 , preferably achromats.
2. Calculate distances t_1 , t_2 .
3. Align lenses 1 and 2 using ruler, mounts, and fine adjustments with fingers.
4. Place lens 3 a distance t_2 from lens 2.
5. Place lens 4 a distance t_1 from lens 3.
6. Check image with camera or eye, and adjust alignment, so background and image match.
7. Improve alignment by iterating steps 3-5, while ensuring lenses are parallel to each other and that their centers align. Check for different angles and reiterate alignment until satisfied.

Notes: This lab/exercise can range anywhere from beginner to advanced, in difficulty level. This depends on how close the image (through all 4 lenses) should match the background (outside of the lenses). Objects can be cloaked (disappear) with minimal alignment, so students with no optics experience can accomplish such “cloaking” effect. To observe a good image requires precise calculation, alignment, and high-quality lenses, which take time, practice, and resources.

Advanced:

- Solve for t_1 , t_2 by equating combined ABCD matrix to cloaking ABCD matrix. For best results, solve for, and use in setup, surface to surface distances, by using thick lens ABCD matrices instead of thin lens matrices.
- Align with laser and ruler: A collimated laser should exit the completed 4 lens system (or the 2 lens sub-systems of lenses 1 & 2, or lenses 3 & 4), still collimated. A laser beam should exit the completed 4 lens system undeviated in position, and unchanged in size (magnification), even when the 4 lenses are rotated a little with respect to the laser.
- Calculate and measure lens surface to surface distances precisely, using manufacturer specifications for lenses and lens mounts. This allows students to measure with a ruler using lens mount edges, and avoid damaging lens surfaces.



Possible Applications

- Some ideas
- Practical uses likely from:
The public,
designers,
entrepreneurs,
industry, artists,
engineers, etc.

