



Paraxial Cloaking- A "Rochester Cloak" [1] Is an Ideal, Broadband and Omnidirectional Cloak Possible? [2]

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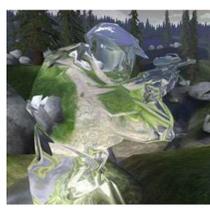


A. Summary

We use 'paraxial' (small-angle) geometric optics to demonstrate a ray optics cloak, after defining invisibility cloaking quantitatively [1]. To our knowledge, our 4 lens design was the first 3D, transmitting cloak that worked for a continuous range of viewing angles, while being broadband for the visible spectrum. We follow with broadband, full-field (phase + amplitude) matching [2].

B. Background

1) **Fiction:** Harry Potter, Star Trek, games, Invisible Man.

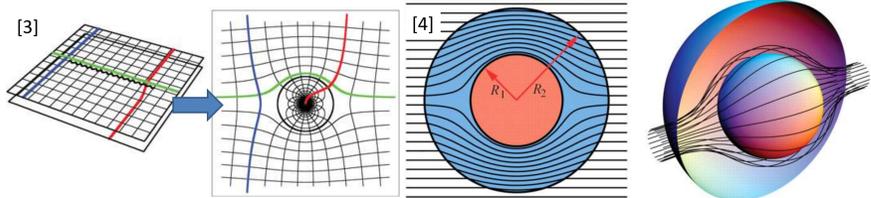


2) **Active invisibility:** Tachi Lab (Japan, 2003), Mercedes-Benz (2012), Land Rover (2014).



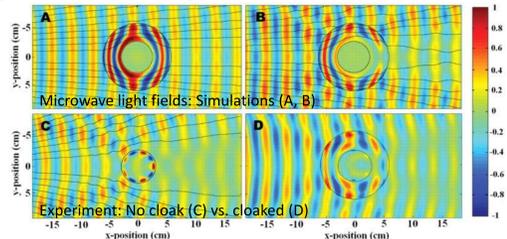
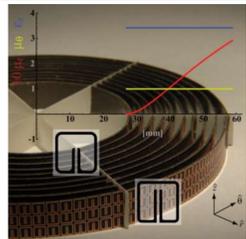
C. Transformation Optics [3,4]

Use coordinate transformations to map a virtual space where no light enters, into physical space where light bends around a real space.



To accomplish this, make 'metamaterials' (artificial materials), that have patterns and electromagnetic properties (ϵ , μ) dictated by the transform.

• **First demonstration (2006)** [5]: Used split-rings for 2D microwave cloaking.



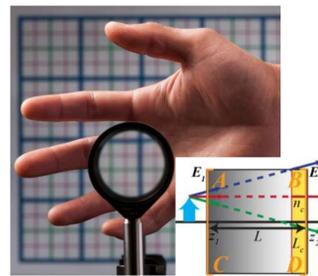
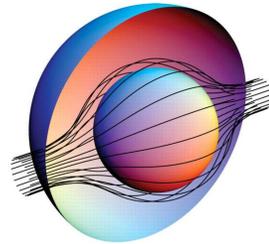
• Great interest in scientific cloaking followed: Temporal event cloaking, thin radio wave canceling cloak, seismic cloaking, 'carpet' cloaking, etc.

D. Define a "Cloak"

• We use the definition "to hide," instead of a wearable clothing.

• A 'perfect' cloak should:

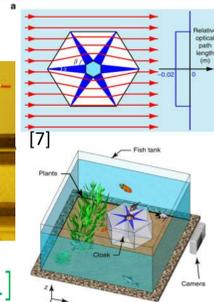
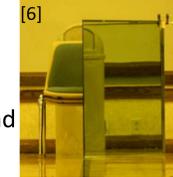
- 1) Cloak a non-zero volume,
- 2) Make itself and the hidden object appear transparent.



'Ideal' Cloak Properties	Transformation Optics	Initial Ray Optics Cloaking	Paraxial Cloaking
Broadband	Difficult	Excellent	Excellent
Visible spectrum			
Isotropic			
Macroscopic			
3D	Some challenges		
Full-field (+phase)	Excellent	~No (1 or discrete freq.)	Broadband (theory)
Omnidirectional		1 or discrete directions	Continuous multidirection

E. Initial Ray Optics Cloaking [6,7]

- Drop phase-matching of light fields to build macroscopic cloaks for visible light.
- Limited to 1 direction, or discrete directions.
- Issues: For change in viewing angles, background shifts, or cloaking region revealed. Also, some do not match magnification with object.



F. Paraxial Ray Optics Cloaking [1]

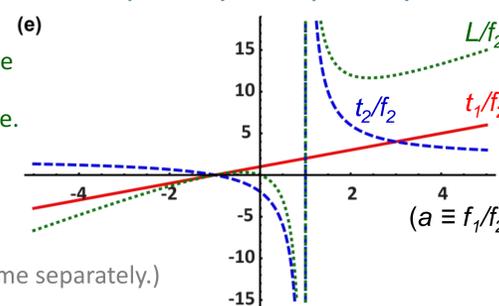
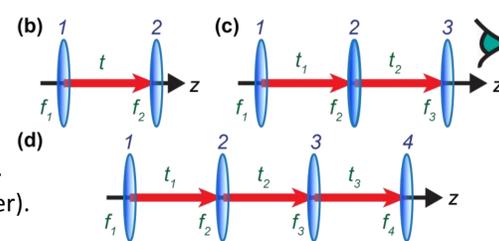
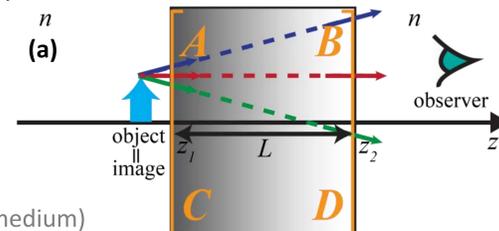
• **Quantitatively define "cloaking":** Using geometric optics, in the small-angle ('paraxial') limit, a cloak is an optical system with ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{perfect cloak}} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}$$

(L = Length of the cloaking system, n = Refractive index of the ambient medium)

G. Solve Cloaking ABCD Matrix

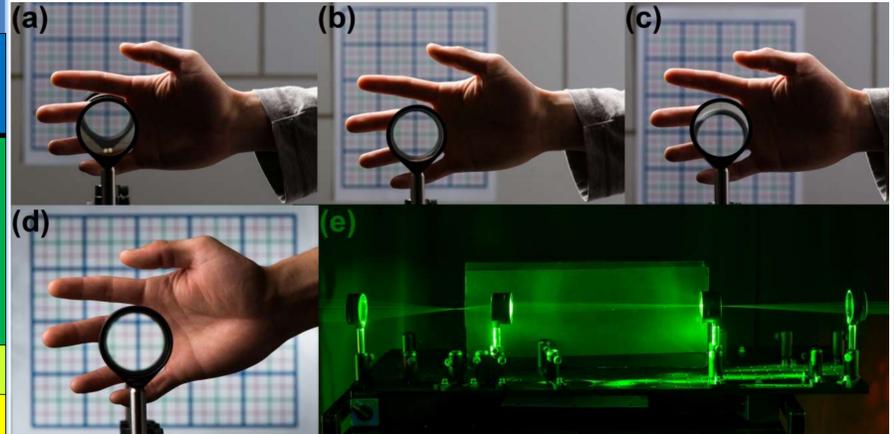
- (a) Find **simplest** ABCD solution: Use rotationally symmetric thin lenses.
- (b) **1 or 2 lens(es):** $f_1=f_2=\pm\infty$ (no power).
- (c) **3 lenses:** Can asymptotically approach 'perfect' cloak for $L \rightarrow 0$.
- (d) **4 lenses:** At least 4 needed to solve 'perfect' cloak ABCD with lenses and have non-zero cloaking volume.
- (e) Figure (e) shows all left-right symmetric ($f_1=f_4, f_2=f_3, t_1=t_3$) 4 lens cloaking solutions*.



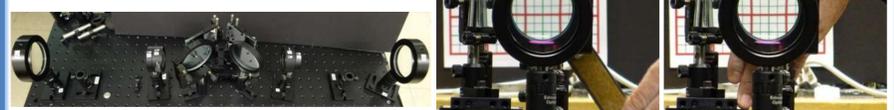
* (Need to check non-zero cloak volume separately.)

H. "Rochester Cloak"

- **4 lens cloak** that solves 'perfect' cloak ABCD matrix with cloakable volume.
- **Advantages:** Broadband for entire visible spectrum, 3D, isotropic, macroscopically scalable, and uses off-the-shelf optics.
- **Limitations:** Center *not* cloaked, $\sim 3^\circ$ field-of-view, edge effects, not full-field.
- **Alignment:** Sensitive. $\sim 1\%$ t_1, t_2, t_3 change causes magnification=1 to $\sim 50\%$.



• **Rochester Cloak version 2:** $\sim 2x$ field-of-view, 1.5x cloaking diameter, center-axis region cloaked.



I. Paraxial Full-Field Cloaking [2]

1) **Field propagation** (Huygens' integral in Fresnel (paraxial) approximation):

$$\tilde{E}_2 = i \frac{e^{-ik_0 L_0}}{B \lambda_0} \iint \tilde{E}_1 \exp \left\{ -i \frac{\pi}{B \lambda_0} [A(x_1^2 + y_1^2) - 2(x_1 x_2 + y_1 y_2) + D(x_2^2 + y_2^2)] \right\} dx_1 dy_1$$

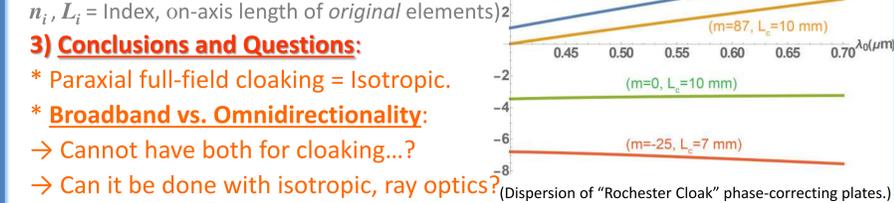
($L_0 = \sum_i n_i L_i$ = On-axis optical path length, (x, y) = Transverse space, λ_0, k_0 = Wavelength, wave vector ($2\pi/\lambda_0$) in vacuum, \tilde{E} = Complex field amplitude, (A, B, C, D) = Matrix coefficients)

2) **Full-field phase-matching:**

Add a thin, flat plate with index n_c and length L_c .

$$n_c(\lambda_0) = n(\lambda_0) + \frac{1}{L_c} \left(m \lambda_0 + \sum_i [n(\lambda_0) - n_i(\lambda_0)] L_i \right)$$

(m = Integer multiple for 2π phase-matching, n_i, L_i = Index, on-axis length of original elements)



3) **Conclusions and Questions:**

* Paraxial full-field cloaking = Isotropic.

* **Broadband vs. Omnidirectionality:**

→ Cannot have both for cloaking...?

→ Can it be done with isotropic, ray optics?

J. References

- [1] J. Choi and J. Howell, *Opt. Express* **22**, 29465 (2014).
- [2] J. Choi and J. Howell, *Opt. Express* **23**, 15857 (2015).
- [3] U. Leonhardt, *Science* **312**, 1777 (2006).
- [4] J. Pendry, D. Schurig, and D. Smith, *Science* **312**, 1780 (2006).
- [5] D. Schurig et al., *Science* **314**, 977 (2006).
- [6] J. C. Howell, J. B. Howell, J. Choi, *Applied Optics* **53**, 1958 (2014).
- [7] H. Chen et al., *Nature Communications* **4**, 2652 (2013).

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